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Numerical approach to solve Navier Stokes equations in Neutrosophic Environment

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Abstract

In this study, a detailed analysis of Navier Stokes equations under uncertainty with no slip conditions by employing neutrosophic numbers is carried out. By employing appropriate similarity transformation, the governing PDE's are transformed into ODE's. By using BVP4C approach, these ODE's are transmuted into neutrosophic differential equations (NDE) by employing $(\bar{A}, \bar{B}, \bar{\Gamma})_{cut}$ approach. The results are presented graphically to analyze the effectiveness of the above method.

Keywords: neutrosophic number, no slip condition, neutrosophic differential equations, triangular neutrosophic number, trapezoidal neutrosophic number.

Introduction

According to Zadeh(1965), the theory of fuzzy set is characterized by the membership function. This idea was expanded upon by Atanassov (1986 and 1999), by adding a non-membership function. This gives rise to a novel idea known as the Intuitionistic Fuzzy Set. Triangular intuitionistic fuzzy sets were later pictured in 2007 by Liu and Yuan by combining triangle fuzzy sets with intuitionistic fuzzy sets. Ye (2014) created trapezoidal Intuitionistic fuzzy sets by substituting trapezoidal fuzzy numbers for triangular fuzzy numbers in both truth and falsity membership functions. However, the objective of indeterminacy is absent from intuitionistic fuzzy logic and traditional fuzzy logic.

Smarandache (1998, 2002) improved the framework beyond fuzzy logic by adding the indeterminacy term to address this flaw. Thus, truth, indeterminacy and false values are the three parts of neutrosophic logic, and membership values are specified inside the non-standard interval $^{-}]0,1[^{+}$. By adding indeterminacy to the fuzzy logic framework, Smarandache (2005) paraded the idea of Neutrosophic sets as a simplified form of intuitionistic fuzzy sets. The description of ambiguity, imprecision, and inconsistency is made possible by this structure.

System of ordinary differential equations of order one represents the rate of change of multiple variables simultaneously. This allows analyzing how these variables interact and evolve overtime with respect to another independent variable. It has many applications in Physics, chemistry, biology, economics and finance, engineering, weather forecasting and social sciences. *Fuzzy differential equations (FDEs)* arrive as a realistic way to model the promulgation of conscious uncertainty in a powerful environment. Bede et al. [9] investigated first order linear FDE's under various interpretations and demonstrated that the behavior of the solutions varies depending on the interpretation used.

Further, to improve upon fuzzy differential equations, intuitionistic fuzzy differential equations were developed. Ettoussi et al. [11] explored how intuitionistic fuzzy differential equations can be solved uniquely and exist using method of successive approximation and discuss the continuity of these solutions in the context of fuzzy set theory. Neutrosophic differential equations allow for a more accurate and realistic modeling of these systems by capturing all facets of uncertainty. Moi et al. [12] has examined second order boundary value problem through neutrosophic differential equation.

From the above mentioned literature survey, researchers used various methods to solve the Navier-Stokes equation. According to my knowledge no research is taken to address the problem using trapezoidal and triangular fuzzy numbers. This research fulfills the gap by solving the problem using BVP4C approach in neutrosophic environment.

2. Preliminaries

In this segment, few preliminary conceptualization of neutrosopic set and some notations are mentioned for better understanding.

Definition 2.1: Universal set

A universal set is the set of all objects or elements under consideration in a specific context. It is indicated by X.

Definition 2.2 : Neutrosophic set(Kamal et. al, 2023)

Let X is a universal set. A Neutrosophic set $(\bar{\mathcal{N}_s})$ on X is defined as $\bar{\mathcal{N}_s} = \{\mathcal{T}_{\bar{\mathcal{N}_s}}(x), \mathcal{J}_{\bar{\mathcal{N}_s}}(x), \mathcal{F}_{\bar{\mathcal{N}_s}}(x) : x \epsilon X\}$, where $\mathcal{T}_{\bar{\mathcal{N}_s}}(x)$, $\mathcal{J}_{\bar{\mathcal{N}_s}}(x)$, $\mathcal{F}_{\bar{\mathcal{N}_s}}(x) : X \longrightarrow {}^{-}]0,1[^{+}$ classifies the degree of uncertainty, degree of hesitation and degree of falseness respectively of the element

$x \epsilon X / \ ^{-}0 \le \mathcal{T}_{\bar{\mathcal{N}_s}}(x) + \mathcal{J}_{\bar{\mathcal{N}_s}}(x) + \mathcal{F}_{\bar{\mathcal{N}_s}}(x) \le 3^{+}$.

Definition 2.3 : $(\bar{A}, \bar{B}, \bar{\Gamma})_{cut}$ of Neutrosophic set (Sumathi and Priya, 2018)

The$(\bar{A}, \bar{B}, \bar{\Gamma})_{cut}$ of Neutrosophic set is defined as $\bar{\mathcal{N}}_{s(\bar{A},\bar{B},\bar{\Gamma})} = \{\langle \mathcal{T}_{\bar{\mathcal{N}_s}}(x), \mathcal{J}_{\bar{\mathcal{N}_s}}(x), \mathcal{F}_{\bar{\mathcal{N}_s}}(x) \rangle : x \epsilon X, \mathcal{T}_{\bar{\mathcal{N}_s}}(x) \ge \bar{A}, \mathcal{J}_{\bar{\mathcal{N}_s}}(x) \ge \bar{B}, \mathcal{F}_{\bar{\mathcal{N}_s}}(x) \ge \bar{\Gamma}\}$, where $\bar{A}, \bar{B}, \bar{\Gamma} \epsilon [0,1]$ such that $0 \le \bar{A} + \bar{B} + \bar{\Gamma} \le 3$.

Definition 2.4: Neutrosophic number (Sumathi and Priya, 2018)

A Neutrosophic number defined on the set of real numbers $\mathbb{R}$ is a Neutrosophic number, if it satisfies the below mentioned properties:

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i. $\overline{\mathcal{N}}_s$ is normal if there exists $x_0 \epsilon \mathbb{R}$:: $\mathcal{T}_{\overline{\mathcal{N}}_s}(x_0)$, $\left(\mathcal{J}_{\overline{\mathcal{N}}_s}(x_0) = \mathcal{F}_{\overline{\mathcal{N}}_s}(x_0) = 0\right)$.

ii. $\overline{\mathcal{N}}_s$ is convex for the truth function $\mathcal{T}_{\overline{\mathcal{N}}_s}(x)$ (ie) $\left(\mathcal{T}_{\overline{\mathcal{N}}_s}(cx_1 + (1-c)x_2) \geq min\left(\mathcal{T}_{\overline{\mathcal{N}}_s}(x_1), \mathcal{T}_{\overline{\mathcal{N}}_s}(x_2)\right)\right)$ for all $x_1, x_2 \in \mathbb{R}$ and $c \in [0,1]$.

iii. $\overline{\mathcal{N}}_s$ is concave for the indeterministic function $\mathcal{J}_{\overline{\mathcal{N}}_s}(x)$ and falsity function $\mathcal{F}_{\overline{\mathcal{N}}_s}(x)$.

$(ie)\ \mathcal{J}_{\overline{\mathcal{N}}_s}\left(cx_1 + (1-c)x_2 \geq max\left(\mathcal{J}_{\overline{\mathcal{N}}_s}(x_1), \mathcal{J}_{\overline{\mathcal{N}}_s}(x_2)\right)\right)$ and $\mathcal{F}_{\overline{\mathcal{N}}_s}\left(cx_1 + (1-c)x_2 \geq max\left(\mathcal{F}_{\overline{\mathcal{N}}_s}(x_1), \mathcal{F}_{\overline{\mathcal{N}}_s}(x_2)\right)\right)$, for all $x_1, x_2 \in \mathbb{R}$ and $c \in [0,1]$.

Definition 2.5 : Triangular Neutrosophic Number (Sumathi and Priya, 2018)

A Triangular Neutrosophic Number is a subset of Neutrosophic set in $\mathbb{R}$ with the following truth function, indeterministic function, falsity function is defined as

$$\mathcal{T}_{\overline{\mathcal{N}}_s}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)\omega_{\overline{\mathcal{N}}_s} & if\ a_1 \leq x \leq a_2 \\ \omega_{\overline{\mathcal{N}}_s} & if\ x = a_2 \\ \left(\frac{a_2-x}{a_3-a_2}\right) & if\ a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

$$\mathcal{T}_{\overline{\mathcal{N}}_s}(x) = \begin{cases} \left(\frac{a_1-x}{a_2-a_1}\right)\delta_{\overline{\mathcal{N}}_s} & if\ a_1 \leq x \leq a_2 \\ \delta_{\overline{\mathcal{N}}_s} & if\ x = a_2 \\ \left(\frac{x-a_3}{a_3-a_2}\right) & if\ a_2 \leq x \leq a_3 \\ 1 & otherwise \end{cases} \qquad (2.1)$$

$$\mathcal{F}_{\overline{\mathcal{N}}_s}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)\epsilon_{\overline{\mathcal{N}}_s} & if\ a_1 \leq x \leq a_2 \\ \epsilon_{\overline{\mathcal{N}}_s} & if\ x = a_2 \\ \left(\frac{x-a_3}{a_3-a_2}\right) & if\ a_2 \leq x \leq a_3 \\ 1 & otherwise \end{cases}$$

Where $^{-}0 \leq \mathcal{T}_{\overline{\mathcal{N}}_s}(x) + \mathcal{J}_{\overline{\mathcal{N}}_s}(x) + \mathcal{F}_{\overline{\mathcal{N}}_s}(x) \leq 3^{+}, x\epsilon\overline{\mathcal{N}}_s$.

Definition 2.6 : $(\bar{A}, \bar{B}, \bar{\Gamma})_{cut}$ of Triangular Neutrosophic Number (Sumathi and Priya, 2018)

The $(\bar{A}, \bar{B}, \bar{\Gamma})_{cut}$ of Triangular Neutrosophic Number = $\langle(a_1, a_2, a_3); \omega_{\overline{\mathcal{N}}_s}, \delta_{\overline{\mathcal{N}}_s}, \epsilon_{\overline{\mathcal{N}}_s}\rangle$ is defined as follows:

$\overline{\mathcal{N}}_{s(\bar{A},\bar{B},\bar{\Gamma})} = \left[\mathcal{T}_{\overline{\mathcal{N}}_s}{}^1(\bar{A}), \mathcal{T}_{\overline{\mathcal{N}}_s}{}^2(\bar{A});\ \mathcal{J}_{\overline{\mathcal{N}}_s}{}^1(\bar{B}), \mathcal{J}_{\overline{\mathcal{N}}_s}{}^2(\bar{B}); \mathcal{F}_{\overline{\mathcal{N}}_s}{}^1(\bar{\Gamma}), \mathcal{F}_{\overline{\mathcal{N}}_s}{}^2(\bar{\Gamma})\right]$, where

$$\mathcal{T}_{\overline{\mathcal{N}}_s}{}^1(\bar{A}) = [a_1 + \bar{A}(a_2 - a_1)]\omega_{\overline{\mathcal{N}}_s}, \mathcal{T}_{\overline{\mathcal{N}}_s}{}^2(\bar{A}) = [a_3 - \bar{A}(a_3 - a_2)]\omega_{\overline{\mathcal{N}}_s}$$
$$\mathcal{J}_{\overline{\mathcal{N}}_s}{}^1(\bar{A}) = [a_2 - \bar{A}(a_2 - a_1)]\delta_{\overline{\mathcal{N}}_s}, \mathcal{J}_{\overline{\mathcal{N}}_s}{}^2(\bar{A}) = [a_2 + \bar{B}(a_3 - a_2)]\delta_{\overline{\mathcal{N}}_s} \qquad (2.2)$$
$$\mathcal{F}_{\overline{\mathcal{N}}_s}{}^1(\bar{A}) = [a_2 - \bar{\Gamma}(a_2 - a_1)]\epsilon_{\overline{\mathcal{N}}_s}, \mathcal{F}_{\overline{\mathcal{N}}_s}{}^2(\bar{A}) = [a_2 + \bar{\Gamma}(a_3 - a_2)]\epsilon_{\overline{\mathcal{N}}_s}$$

Here, $0 \leq \bar{A} \leq 1$, $0 \leq \bar{B} \leq 1$, $0 \leq \bar{\Gamma} \leq 1$ and $^{-}0 \leq \bar{A} + \bar{B} + \bar{\Gamma} \leq 3^{+}$ which is shown in figure 1.

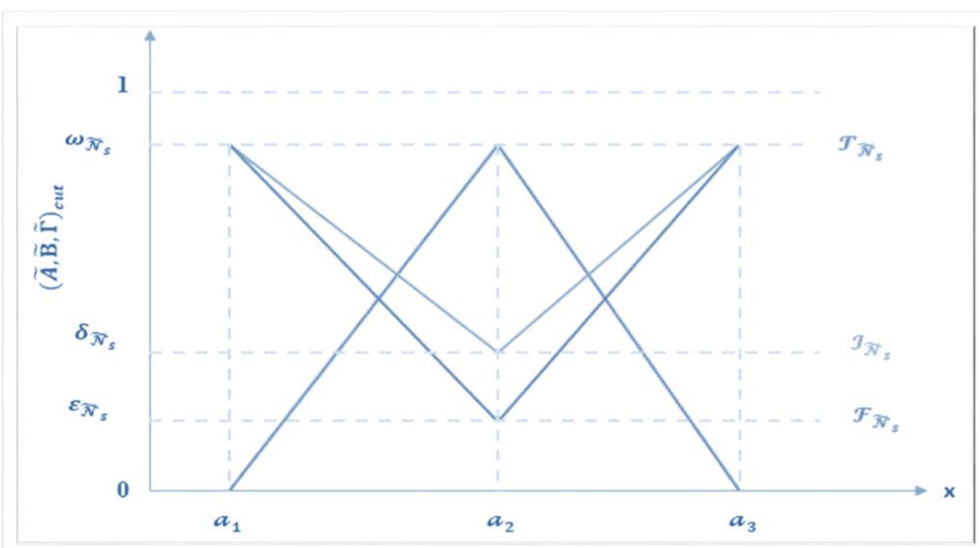

Figure 1. Membership function of Triangular Neutrosophic number

Definition 2.7 : Trapezoidal Neutrosophic Number (Sumathi and Sweety, 2019)

A Trapezoidal Neutrosophic Number is a subset of $\bar{\mathcal{N}}_s$ in $\mathbb{R}$ with the following truth function, indeterministic function, falsity function is defined as

$$\mathcal{T}_{\bar{\mathcal{N}}_s}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)\omega_{\bar{\mathcal{N}}_s} & if\ a_1 \le x \le a_2 \\ \omega_{\bar{\mathcal{N}}_s} & if\ a_2 \le x \le a_3 \\ \left(\frac{a_4-x}{a_4-a_3}\right) & if\ a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

$$\mathcal{T}_{\bar{\mathcal{N}}_s}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)\delta_{\bar{\mathcal{N}}_s} & if\ a_1 \le x \le a_2 \\ \delta_{\bar{\mathcal{N}}_s} & if\ a_2 \le x \le a_3 \\ \left(\frac{a_4-x}{a_4-a_3}\right) & if\ a_3 \le x \le a_4 \\ 1 & otherwise \end{cases} \qquad (2.3)$$

$$\mathcal{F}_{\bar{\mathcal{N}}_s}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)\epsilon_{\bar{\mathcal{N}}_s} & if\ a_1 \le x \le a_2 \\ \epsilon_{\bar{\mathcal{N}}_s} & if\ a_2 \le x \le a_3 \\ \left(\frac{a_4-x}{a_4-a_3}\right) & if\ a_3 \le x \le a_4 \\ 1 & otherwise \end{cases}$$

Where $^{-}0 \le \mathcal{T}_{\bar{\mathcal{N}}_s}(x) + \mathcal{J}_{\bar{\mathcal{N}}_s}(x) + \mathcal{F}_{\bar{\mathcal{N}}_s}(x) \le 3^{+}, x\epsilon\bar{\mathcal{N}}_s$.

Definition 2.8: $(\bar{A},\bar{B},\bar{\Gamma})_{cut}$ of Trapezoidal Neutrosophic Number(Sumathi and Sweety, 2019)

The $(\bar{A},\bar{B},\bar{\Gamma})_{cut}$ of Trapezoidal Neutrosophic Number = $\langle(a_1,\ a_2,\ a_3,\ a_4); \omega_{\bar{\mathcal{N}}_s}, \delta_{\bar{\mathcal{N}}_s}, \epsilon_{\bar{\mathcal{N}}_s}\rangle$ is defined as follows:

$\bar{\mathcal{N}}_{s(\bar{A},\bar{B},\bar{\Gamma})} = \left[\mathcal{T}_{\bar{\mathcal{N}}_s}{}^{1}(\bar{A}), \mathcal{T}_{\bar{\mathcal{N}}_s}{}^{2}(\bar{A});\ \mathcal{J}_{\bar{\mathcal{N}}_s}{}^{1}(\bar{B}), \mathcal{J}_{\bar{\mathcal{N}}_s}{}^{2}(\bar{B}); \mathcal{F}_{\bar{\mathcal{N}}_s}{}^{1}(\bar{\Gamma}), \mathcal{F}_{\bar{\mathcal{N}}_s}{}^{2}(\bar{\Gamma})\right]$, where

$$\mathcal{T}_{\bar{\mathcal{N}}_s}{}^{1}(\bar{A}) = [a_1 + \bar{A}(a_2 - a_1)]\omega_{\bar{\mathcal{N}}_s},\ \mathcal{T}_{\bar{\mathcal{N}}_s}{}^{2}(\bar{A}) = [a_4 - \bar{A}(a_3 - a_2)]\omega_{\bar{\mathcal{N}}_s}$$
$$\mathcal{J}_{\bar{\mathcal{N}}_s}{}^{1}(\bar{A}) = [a_2 - \bar{A}(a_2 - a_1)]\delta_{\bar{\mathcal{N}}_s},\ \mathcal{J}_{\bar{\mathcal{N}}_s}{}^{2}(\bar{A}) = [2 + \bar{B}(a_3 - a_2)]\delta_{\bar{\mathcal{N}}_s} \qquad (2.4)$$
$$\mathcal{F}_{\bar{\mathcal{N}}_s}{}^{1}(\bar{A}) = [a_2 - \bar{\Gamma}(a_2 - a_1)]\epsilon_{\bar{\mathcal{N}}_s},\ \mathcal{F}_{\bar{\mathcal{N}}_s}{}^{2}(\bar{A}) = [a_3 + \bar{\Gamma}(a_3 - a_2)]\epsilon_{\bar{\mathcal{N}}_s}$$

Here, $0 \le \bar{A} \le 1$, $0 \le \bar{B} \le 1$, $0 \le \bar{\Gamma} \le 1$ and $^{-}0 \le \bar{A} + \bar{B} + \bar{\Gamma} \le 3^{+}$ which is shown in figure 2.

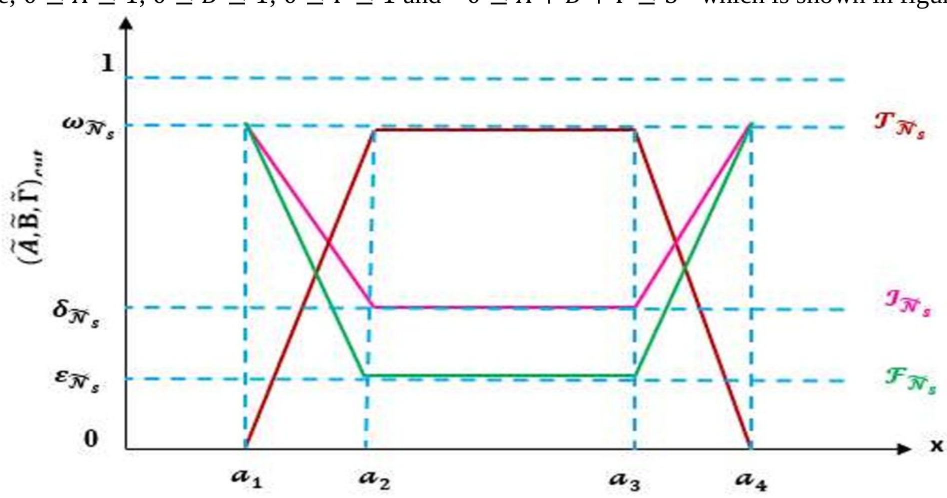

Figure 2: Membership function of Trapezoidal Neutrosophic Number

Definition 2.9 : Strong and Weak solution (Sumathi and Sweety, 2019)

Let the solution of the Neutrosophic differential equation be $y(x)$ and its $(\bar{A},\bar{B},\bar{\Gamma})_{cut}$ be $[y_1(x,\bar{A}), y_2(x,\bar{A}), y_1{}'(x,\bar{B}), y_2{}'(x,\bar{B}), y_1{}''(x,\bar{\Gamma}), y_2{}''(x,\bar{\Gamma})]$. The solution is string if

i. $\frac{dy_1(x,\bar{A})}{d\bar{A}} > 0, \frac{dy_2(x,\bar{A})}{d\bar{A}} < 0\ \forall\ \bar{A}\ \in [0,1], y_1(x,1) \le y_2(x,1).$

ii. $\frac{dy_1{}'(x,\bar{B})}{d\bar{B}} > 0, \frac{dy_1{}'(x,\bar{B})}{d\bar{B}} < 0\ \forall\ \bar{A}\ \in [0,1], y_1{}'(x,0) \le y_2{}'(x,0).$ (2.5)

iii. $\frac{dy_1{}''(x,\bar{\Gamma})}{d,\bar{\Gamma}} > 0, \frac{dy_2{}''(x,\bar{\Gamma})}{d,\bar{\Gamma}} < 0\ \forall\ \bar{A}\ \in [0,1], y_1{}''(x,0) \le y_2{}''(x,0).$

If not, the solution is a weak one.

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3. Governing equations

Consider the steady, incompressible, two-dimensional boundary layer flow across a static wedge. Figure 3. shows the geometrical coordinates and the physical model. We take into account the consequences of the no-slip condition and suppose that the free stream's velocity is $u_e(x) = Ux^m$. Look at a Cartesian coordinate system (x, y), where x and y are the coordinates measured normal to the wedge and along its surface, respectively. The partial differential equations governing the problem with the boundary conditions under the aforementioned assumptions are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial x^2} - \frac{\sigma {B_0}^2}{\rho}(u - U) \quad (3.2)$$

With boundary conditions

$$u = 0, v = 0 \qquad \text{at } y = 0$$
$$u(x, y) = u_e(x) \ \text{ as } y \longrightarrow \infty \quad (3.3)$$

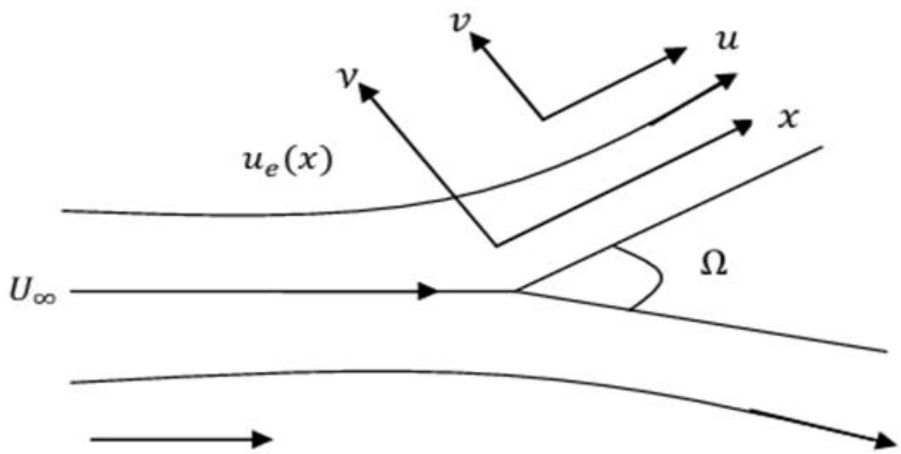

Figure 3: Physical model and coordinate system

Here, the velocity components in the x and y directions are denoted by u and v, respectively. B_0 is the magnetic strength, ν is the kinematic fluid viscosity, ρ is the fluid density, σ is the electrical conductivity and U is the external velocity at the boundary layer's edge.

We introduce the similarity variable ς and stream function $\chi(x, y)$ as defined in Khan et al (2021).

$$\varsigma = \sqrt{\frac{c(m+1)}{2\nu_f}}\, y \text{ and } \chi(x, y) = \sqrt{\frac{2c\nu_f}{m+1}}\, x^{m+1} f(\varsigma) \quad (3.4)$$

Here, $f(\varsigma)$ is the dimensionless stream function. The corresponding velocity components can be expressed as

$$u = \frac{\partial \chi}{\partial y} = cx^m f'(\varsigma),\ v = -\frac{\partial \chi}{\partial x} = \sqrt{\frac{-\nu_f c(m+1)x^{m-1}}{2}}\left[\varsigma f'(\varsigma)\frac{(m-1)}{(m+1)} + f(\varsigma)\right] \quad (3.5)$$

By substituting the above in the boundary layer equations (3.1) and (3.2), we obtain the transformed equation as

$$f'''(\varsigma) \quad + f'(\varsigma)f(\varsigma) + \beta\left[1 - \left(f'(\varsigma)\right)^2\right] + M^2\left(1 - f'(\varsigma)\right) = 0 \quad (3.6)$$

Where $\beta = \frac{2m}{m+1}$, m is related to external velocity and $U(x) \propto x^m$ and $M^2 = \frac{\sigma {B_0}^2}{\rho M U^2}$

The appropriate boundary conditions are

$$f(\varsigma) = 0, \quad f'(\varsigma) = 0 \text{ at } \varsigma = 0$$
$$f'(\varsigma) \longrightarrow 1 \qquad \text{as } \varsigma \longrightarrow \infty \quad (3.7)$$

By using the above equation (3.6), the outcome of pressure gradients on the boundary layer flow can be analyzed. Positive β $(m > 0)$corresponds to accelerating flow, while negative β $(m < 0)$ corresponds to decelerating flow.

4. Numerical Technique

BVP4C approach is adopted to solve the third order boundary value problem(BVP) by converting into initial value problem(IVP). The method involves guessing the initial conditions for the IVP at $\varsigma = 0$, integrating the first order ODE's and adjusting the guessed initial conditions to satisfy the boundary conditions at the end point $\varsigma \longrightarrow \infty$.

Equation (3.6) is transformed into first order ODE's in the form

$$\begin{pmatrix} {y_1}' \\ {y_2}' \\ {y_3}' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ -y_1 y_2 - \beta(1 - {y_2}^2) - M^2(1 - y_2) \end{pmatrix} \quad (4.1)$$

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with corresponding boundary conditions

$$y_1(\varsigma) = y_2(\varsigma) = 0 \quad \text{at } \varsigma = 0$$
$$y_2(\varsigma) \longrightarrow 1 \quad \text{as } \varsigma \longrightarrow \infty \tag{4.2}$$

5. Boundary layer equation in Neutrosophic Environment

Under the Neutrosophic context, the function $f(\varsigma)$ and its derivatives can be expressed as three components each; $\bar{A}$(truth), $\bar{B}$(indeterminacy) and $\bar{\Gamma}$(falsity). Hence, $f(\varsigma)$ can be written as

$$f(\varsigma) = \left(f(\varsigma, \bar{A}), f(\varsigma, \bar{B}), f(\varsigma, \bar{\Gamma})\right) \tag{5.1}$$

Hence, for the function $f(\varsigma)$ and its derivatives, the three corresponding components must be considered. The Neutrosophic form of the boundary layer equation can be written as

$$f'''(\varsigma,\bar{A}) + f'(\varsigma,\bar{A})f(\varsigma,\bar{A}) + \beta^{\bar{A}}\left[1 - \left(f'(\varsigma,\bar{A})\right)^2\right] + M^2\left(1 - f'(\varsigma,\bar{A})\right) = 0$$
$$f'''(\varsigma,\bar{B}) + f'(\varsigma,\bar{B})f(\varsigma,\bar{B}) + \beta^{\bar{B}}\left[1 - \left(f'(\varsigma,\bar{B})\right)^2\right] + M^2\left(1 - f'(\varsigma,\bar{B})\right) = 0 \tag{5.2}$$
$$f'''(\varsigma,\bar{\Gamma}) + f'(\varsigma,\bar{\Gamma})f(\varsigma,\bar{\Gamma}) + \beta^{\bar{\Gamma}}\left[1 - \left(f'(\varsigma,\bar{\Gamma})\right)^2\right] + M^2\left(1 - f'(\varsigma,\bar{\Gamma})\right) = 0$$

With transformed boundary condition

$$f(\varsigma,\bar{A}) = \bar{0},\ f'(\varsigma,\bar{A}) = \bar{0} \quad \text{at } \varsigma = 0 \quad f'(\varsigma,\bar{A}) \longrightarrow 1 \quad \text{as } \varsigma \longrightarrow \infty$$
$$f(\varsigma,\bar{B}) = \bar{0},\ f'(\varsigma,\bar{B}) = \bar{0} \quad \text{at } \varsigma = 0 \quad f'(\varsigma,\bar{B}) \longrightarrow 1 \quad \text{as } \varsigma \longrightarrow \infty \tag{5.3}$$
$$f(\varsigma,\bar{\Gamma}) = \bar{0},\ f'(\varsigma,\bar{\Gamma}) = \bar{0} \quad \text{at } \varsigma = 0 \quad f'(\varsigma,\bar{\Gamma}) \longrightarrow 1 \quad \text{as } \varsigma \longrightarrow \infty$$

The neutrosophic velocity profile is defined as $f'(\varsigma,\bar{A}) = \left[\underline{f'}(\varsigma,\bar{A}), \overline{f'}(\varsigma,\bar{A})\right]$, $0 \le \bar{A} \le 1$. The term $\underline{f'}(\varsigma,\bar{A})$ represents the lower bound and $\overline{f'}(\varsigma,\bar{A})$ represents the upper bound of the neutrosophic velocity, respectively for $\bar{A}$(truth). In the same way, $f'(\varsigma,\bar{B}) = \left[\underline{f'}(\varsigma,\bar{B}), \overline{f'}(\varsigma,\bar{B})\right]$, $0 \le \bar{B} \le 1$, $f'(\varsigma,\bar{\Gamma}) = \left[\underline{f'}(\varsigma,\bar{\Gamma}), \overline{f'}(\varsigma,\bar{\Gamma})\right]$, $0 \le \bar{\Gamma} \le 1$ for $\bar{B}$(indeterminacy) and $\bar{\Gamma}$(falsity), respectively and $^{-}0 \le \bar{A} + \bar{B} + \bar{\Gamma} \le 3^{+}$.

The Triangular neutrosophic number and Trapezoidal neutrosophic number are transformed using $(\bar{A}, \bar{B}, \bar{\Gamma})_{cut}$ is shown in Table 1 for wedge angle parameter β.

Table 1. Parametric form of Triangular neutrosophic number and trapezoidal neutrosophic number for various β.

	Crisp value	Neutrosophic number	Alpha cut approach	Beta cut approach	Gamma cut approach
Triangular Neutrosophic number	[0.01-0.5]	[0.1, 0.3, 0.5; 0.5, 0.7, 0.3]	[(0.1+0.2 $\bar{A}$)0.5, (0.5-0.2$\bar{A}$)0.5]	[(0.3-0.2$\bar{B}$)0.7, (0.3+0.2$\bar{B}$)0.7]	[(0.3-0.2$\bar{\Gamma}$)0.3, (0.3+0.2$\bar{\Gamma}$)0.3]
Trapezoidal neutrosophic number	[0.01-0.5]	[0.1, 0.3, 0.5, 0.7;0.5,0.7,0.3]	[(0.1+0.2$\bar{A}$)0.5, (0.7-0.2$\bar{A}$)0.5]	[(0.3-0.2$\bar{B}$)0.7, (0.5+0.2$\bar{B}$)0.7]	[(0.3-0.2$\bar{\Gamma}$)0.3, (0.5+0.2$\bar{\Gamma}$)0.3]

In the same way, the Triangular neutrosophic number and Trapezoidal neutrosophic number are transformed using $(\bar{A}, \bar{B}, \bar{\Gamma})_{cut}$ is shown in Table 2 for no slip conditions.

Table 2. Parametric form of triangular neutrosophic number and trapezoidal neutrosophic number for no slip conditions.

	No slip conditions	Crisp value	Neutrosophic number	Alpha cut approach	Beta cut approach	Gamma cut approach
Triangular Neutrosophic number	$f(\varsigma) = \bar{0}$	[0.01-0.5]	[0.1, 0.3, 0.5; 0.5, 0.7, 0.3]	[(0.1+0.2$\bar{A}$)0.5, (0.5-0.2$\bar{A}$)0.5]	[(0.3-0.2$\bar{B}$)0.7, (0.3+0.2$\bar{B}$)0.7]	[(0.3-0.2$\bar{\Gamma}$))0.3, (0.3+0.2$\bar{\Gamma}$))0.3]
	$f'(\varsigma) = \bar{0}$	[0.01-0.5]	[0.2, 0.3, 0.6; 0.7, 0.3, 0.5]	[(0.2+0.1$\bar{A}$)0.7, (0.6-0.3$\bar{A}$)0.7]	[(0.3-0.1$\bar{B}$)0.3, (0.3+0.3$\bar{B}$)0.3]	[(0.3-0.1$\bar{\Gamma}$))0.5, (0.3+0.3$\bar{\Gamma}$))0.5]
Trapezoidal neutrosophic number	$f(\varsigma) = \bar{0}$	[0.01-0.5]	[0.1,0.3,0.5, 0.7;0.2,0.4,0.6]	[(0.1+0.2$\bar{A}$)0.2, (0.7-0.2$\bar{A}$)0.2]	[(0.3-0.2$\bar{B}$)0.4, (0.5+0.2$\bar{B}$)0.4]	[(0.3-0.2$\bar{\Gamma}$))0.6, (0.5+0.2$\bar{\Gamma}$))0.6]
	$f'(\varsigma) = \bar{0}$	[0.01-0.5]	[0.1,0.2,0.3, 0.4;0.5,0.3,0.7]	[(0.1+0.1$\bar{A}$)0.5, (0.4-0.1$\bar{A}$)0.5]	[(0.2-0.1$\bar{B}$)0.3, (0.3+0.1$\bar{B}$)0.3]	[(0.2-0.1$\bar{\Gamma}$))0.7, (0.3+0.1$\bar{\Gamma}$))0.7]

5. Validation

To validate the correctness of the BVP4C approach, the results of $f''(0)$ was compared to with the existing results.

Table 3: Comparison of $f''(0)$ for various β (M = 0)

β	Zhang et al. (2009)	Salama (2004)	Present (BVP4C approach)
-0.15	0.216362	0.216362	0.210243
-0.1	0.319270	0.319270	0.328642
0	0.469600	0.469600	0.469230
0.5	0.927680	0.927680	0.925463
1	1.232587	1.232588	1.329624

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6. Result and discussion

The neutrosophic analysis of boundary layer equation is carried out in this section. The uncertainty of wedge angle parameter β and no slip conditions are considered as Triangular and Trapezoidal Neutrosophic numbers. The governing equation are transformed into neutrosophic differential equations (NDE's), numerically solved using BVP4C approach.

6.1 Profile graphs by BVP4C approach

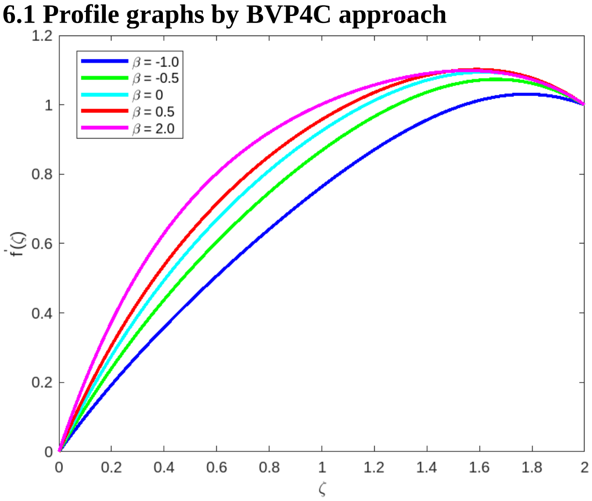

Figure 4. Behavior of $f'(\varsigma)$ against β

Figure 4 demonstrates the velocity distribution against β over the boundary layer. From the figure, it is observed that the wedge angle parameter β controls the boundary layer characteristics. An observation from the figure reveals the fact that velocity at the boundaries increases with the rise in β.

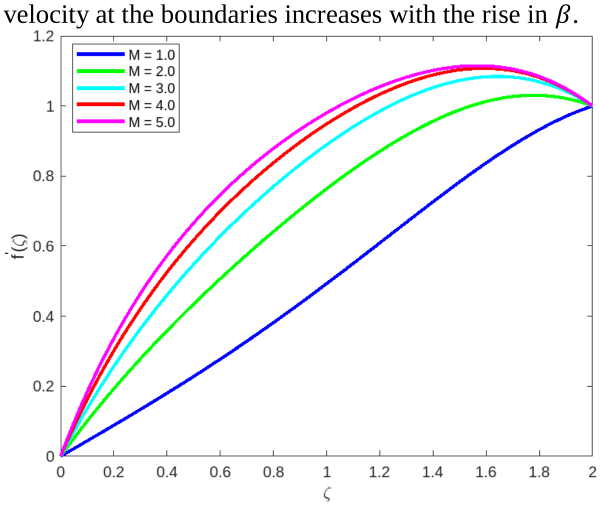

Figure 5(a). Behavior of $f'(\varsigma)$ against M $(\beta < 0)$

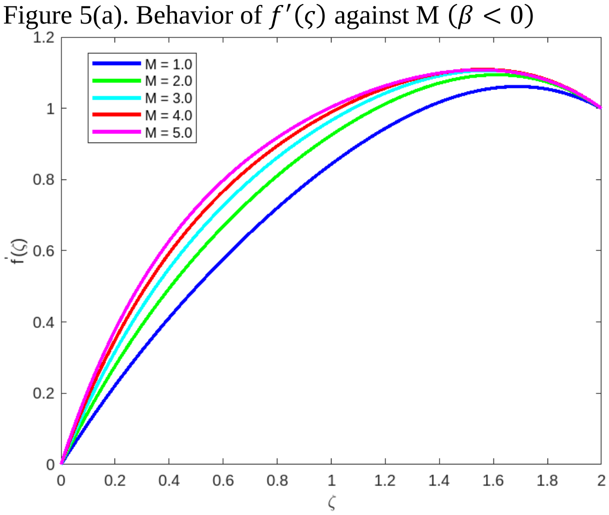

Figure 5(b). Behavior of $f'(\varsigma)$ against M $(\beta = 0)$

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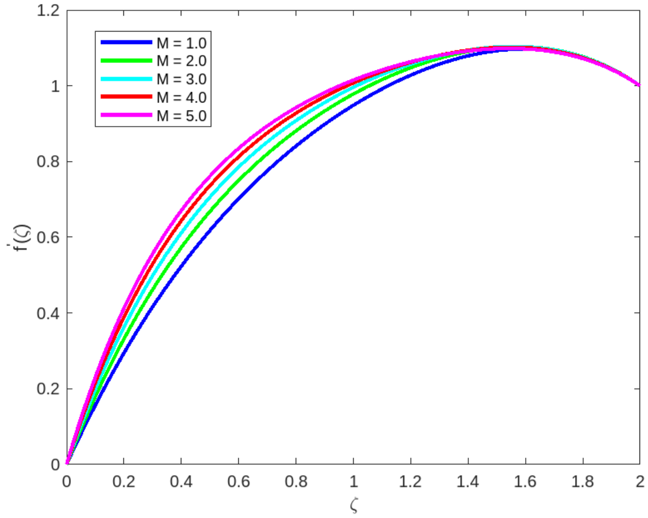

Figure 5(c). Behavior of $f'(\varsigma)$ against M ($\beta > 0$)

The behavior of Magnetic parameter on velocity at the boundary layer is publicized in Figure 5(a) – (c). Figure 5(a) demonstrates the decelerating velocity profile against Magnetic parameter. Figure 5(b) reveals the impression of magnetic parameter over a flat plate. Whilst Fig 5(c) explores the impact of Magnetic parameter on the velocity profile. From the above figures, it is observed that increment in Magnetic parameter boosts the velocity profile at the boundary.

6.2 Neutrosophic Analysis using BVP4C approach

Case 1: Wedge angle parameter β as neutrosophic number

The wedge angle parameter β is considered as triangular neutrosophic number [0.1, 0.3, 0.5; 0.5, 0.7, 0.3] and trapezoidal neutrosophic neutrosophic number [0.1, 0.3, 0.5, 0.7; 0.5, 0.7, 0.3] in the boundary layer equation.

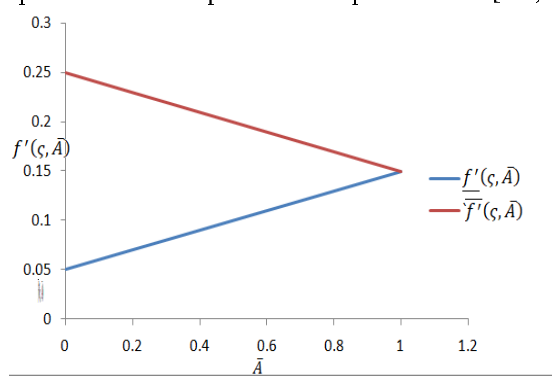

Figure 6(a). Triangular Neutrosophic velocity for truth function $\bar{A}$

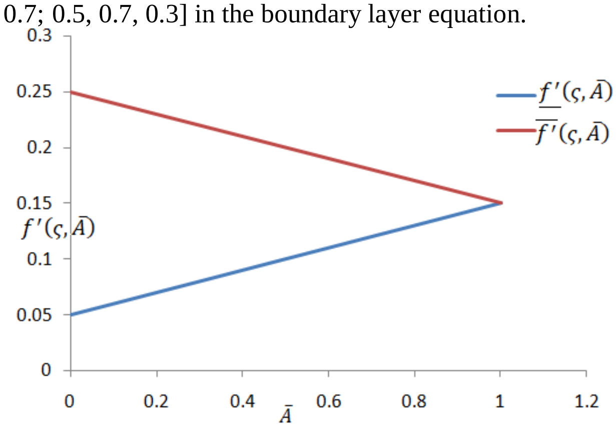

Figure 6(b). Trapezoidal Neutrosophic velocity for truth function $\bar{A}$

Figure 6(a) and 6(b) presents the neutrosophic velocity profile $f'(\varsigma,\bar{A})$ corresponding to the truth component of neutrosophic number. From the figure, we can understand that, $\underline{f'}(\varsigma,\bar{A})$ is exactly increasing and $\overline{f'}(\varsigma,\bar{A})$ is exactly decreasing for all $\bar{A}\epsilon[0,1]$ and $\underline{f'}(\varsigma,\bar{A}) \leq \overline{f'}(\varsigma,\bar{A})$ for both triangular and trapezoidal neutrosophic number.

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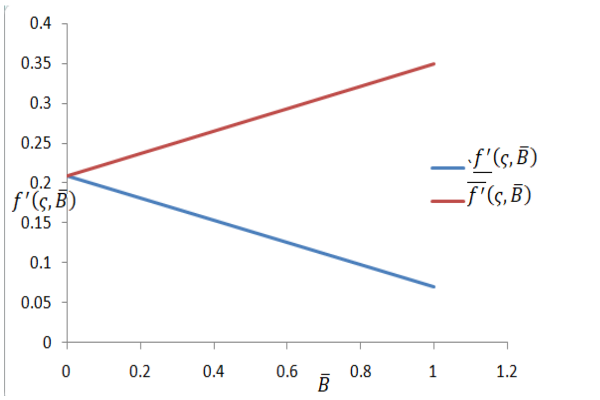

Figure 7(a). Triangular Neutrosophic velocity for indeterministic function $\bar{B}$

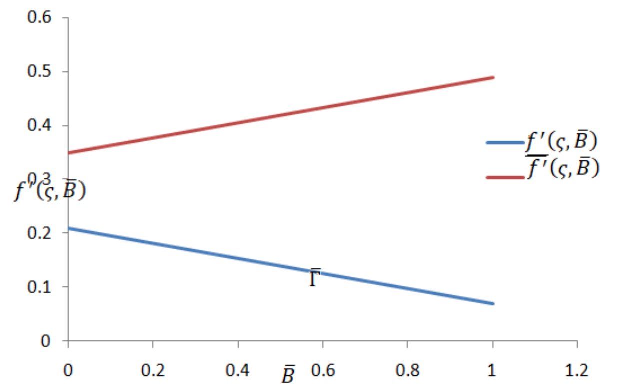

Figure 7(b). Trapezoidal Neutrosophic velocity for indeterministic function $\bar{B}$

Figure 7(a) and 7(b) presents the neutrosophic velocity profile $f'(\varsigma,\bar{B})$ corresponding to the indeterminacy component of neutrosophic number. From the figure, we can conclude that, $\underline{f'}(\varsigma,\bar{B})$ is strictly decreasing and $\overline{f'}(\varsigma,\bar{B})$ is strictly increasing for all $\bar{B}\epsilon[0,1]$ and $\underline{f'}(\varsigma,\bar{B}) \leq \overline{f'}(\varsigma,\bar{B})$ for both triangular and trapezoidal neutrosophic number.

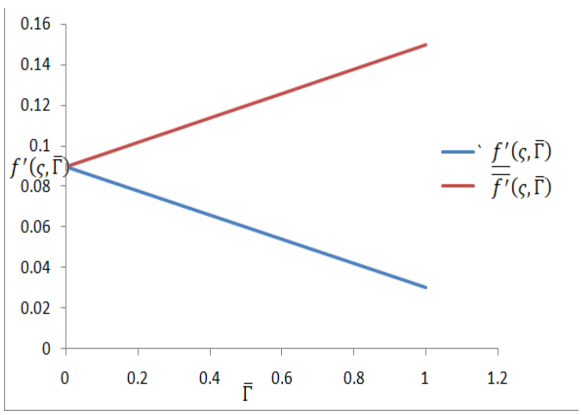

Figure 8(a). Triangular Neutrosophic velocity for falsity function $\bar{\Gamma}$

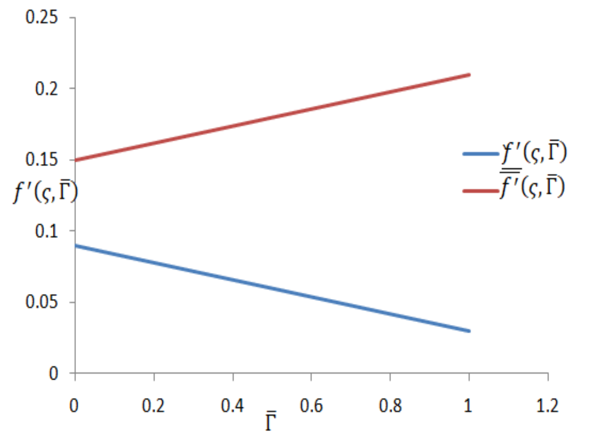

Figure 8(b). Trapezoidal Neutrosophic velocity for falsity function $\bar{\Gamma}$

Figure 8(a) and 8(b) presents the neutrosophic velocity profile $f'(\varsigma,\bar{\Gamma})$ associated with falsity. It depicts that, $\underline{f'}(\varsigma,\bar{\Gamma})$ is strictly decreasing and $\overline{f'}(\varsigma,\bar{\Gamma})$ is strictly increasing for all $\bar{\Gamma}\epsilon[0,1]$ and $\underline{f'}(\varsigma,\bar{\Gamma}) \leq \overline{f'}(\varsigma,\bar{\Gamma})$ for both triangular and trapezoidal neutrosophic number. Hence we can conclude that, the solution of (3.6), $f'(\varsigma,\bar{A},\bar{B},\bar{\Gamma})$ is a strong neutrosophic solution.

Case 2: No slip condition as neutrosophic number

The no slip condition $f(0)$ is considered as triangular neutrosophic number [0.1, 0.3, 0.5; 0.5, 0.7, 0.3] and trapezoidal neutrosophic neutrosophic number [0.1,0.3,0.5, 0.7;0.2,0.4,0.6] in the boundary layer equation.

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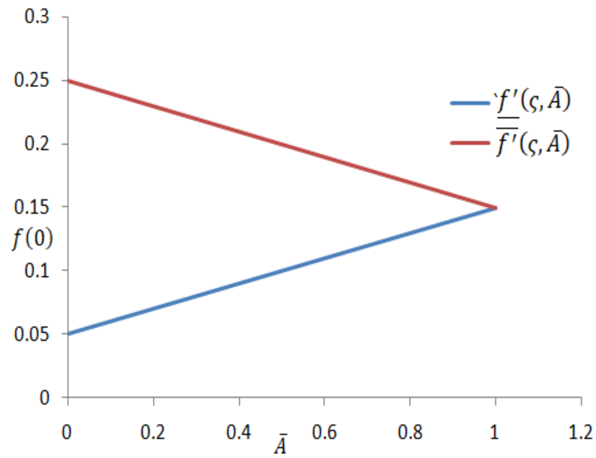

Figure 9(a). Triangular Neutrosophic velocity for $f(0)$

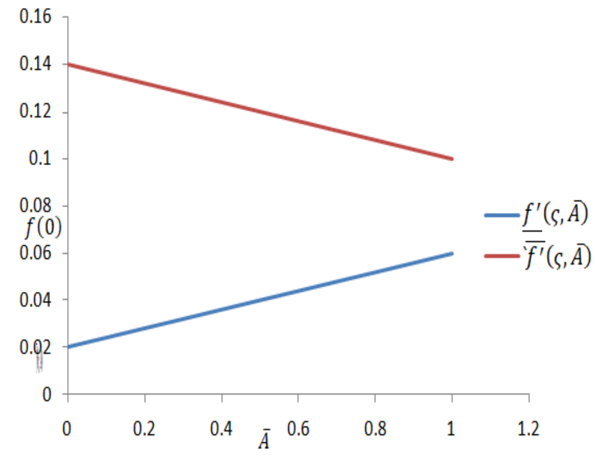

Figure 9(b). Trapezoidal Neutrosophic velocity for $f(0)$

From Figures 9(a) and 9(b), we see that, $\underline{f'}(\varsigma,\bar{A})$ is increasing and $\overline{f'}(\varsigma,\bar{A})$ is strictly decreasing for all $\bar{A}\epsilon[0,1]$, $\underline{f'}(\varsigma,\bar{A}) \leq \overline{f'}(\varsigma,\bar{A})$ for both triangular and trapezoidal neutrosophic number.

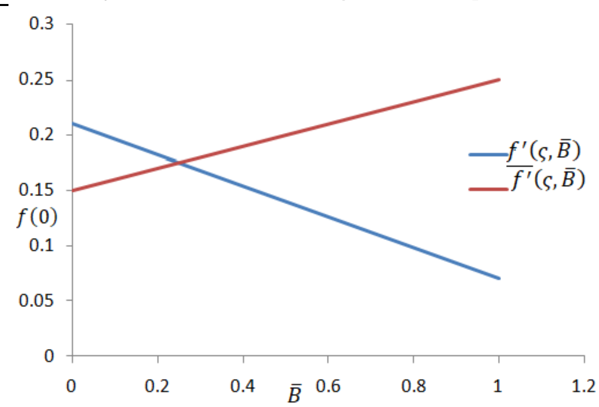

Figure 10(a). Triangular Neutrosophic velocity for $f(0)$

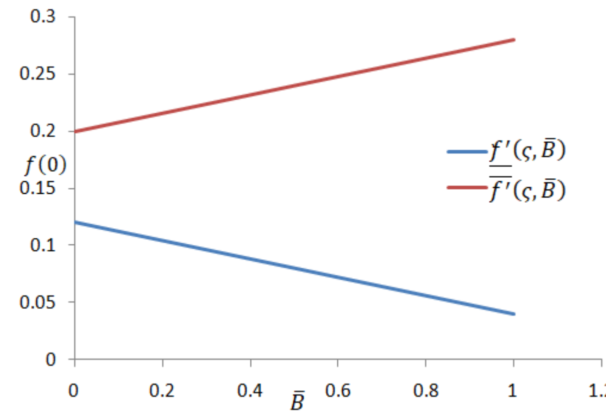

Figure 10(b). Trapezoidal Neutrosophic velocity for $f(0)$

From Figures 10(a) and 10(b), it express that, $\underline{f'}(\varsigma,\bar{B})$ is decreasing and $\overline{f'}(\varsigma,\bar{B})$ is strictly increasing for all $\bar{B}\epsilon[0,1]$, $\underline{f'}(\varsigma,\bar{B}) \leq \overline{f'}(\varsigma,\bar{B})$ for both triangular and trapezoidal neutrosophic number.

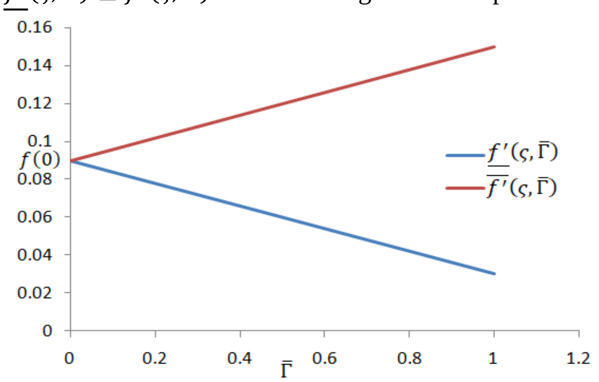

Figure 11(a). Triangular Neutrosophic velocity for $f(0)$

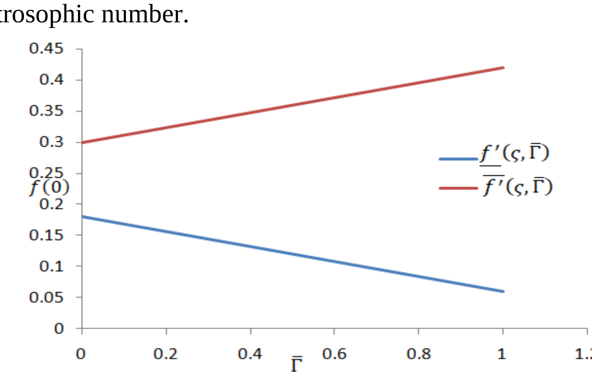

Figure 11(b). Trapezoidal Neutrosophic velocity for $f(0)$

From Figures 11(a) and 11(b), we see that, $\underline{f'}(\varsigma,\bar{\Gamma})$ decreases and $\overline{f'}(\varsigma,\bar{\Gamma})$ strictly increases for all $\bar{\Gamma}\epsilon[0,1]$, $\underline{f'}(\varsigma,\bar{\Gamma}) \leq \overline{f'}(\varsigma,\bar{\Gamma})$ for both triangular and trapezoidal neutrosophic number. From the above discussion, we can conclude that, the solution of (3.6), $f'(\varsigma,\bar{A},\bar{B},\bar{\Gamma})$ is a strong neutrosophic solution.

The no slip condition $f'(0)$ is considered as triangular neutrosophic number [0.2, 0.3, 0.6; 0.7, 0.3, 0.5] and trapezoidal neutrosophic neutrosophic number [0.1,0.2,0.3, 0.4;0.5,0.3,0.7]in the boundary layer equation.

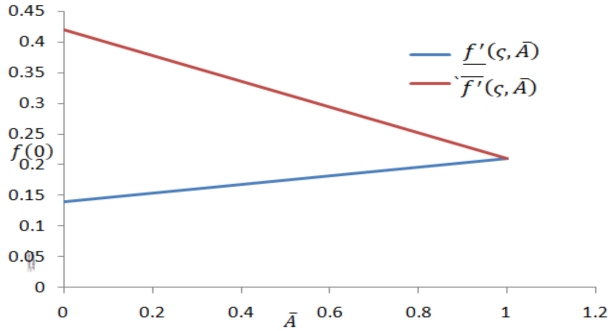

Figure 12(a). Triangular Neutrosophic velocity for $f'(0)$

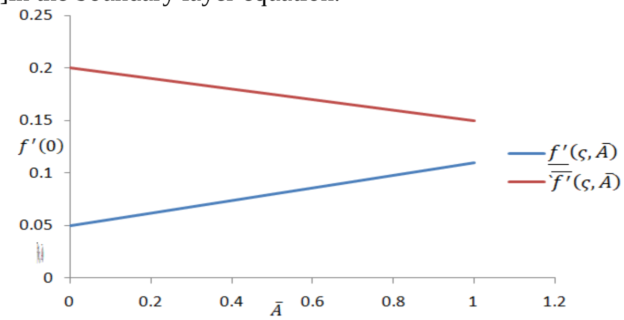

Figure 12(b). Trapezoidal Neutrosophic velocity for $f'(0)$

From Figures 12(a) and 12(b), we see that, $\underline{f'}(\varsigma,\bar{A})$ increases and $\overline{f'}(\varsigma,\bar{A})$ strictly decreases for all $\bar{A}\epsilon[0,1]$, $\underline{f'}(\varsigma,\bar{A}) \leq \overline{f'}(\varsigma,\bar{A})$ for both triangular and trapezoidal neutrosophic number.

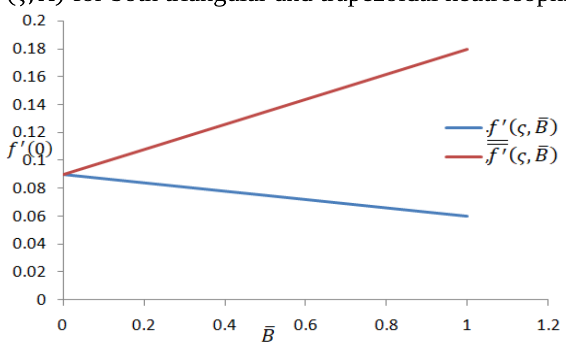

Figure 13(a). Triangular Neutrosophic velocity for $f'(0)$

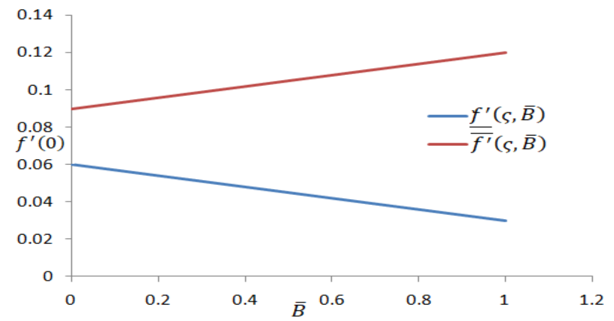

Figure 13(b). Trapezoidal Neutrosophic velocity for $f'(0)$

From Figures 13(a) and 13(b), we see that, $\underline{f'}(\varsigma,\bar{B})$ decreases and $\overline{f'}(\varsigma,\bar{B})$ strictly increases for all $\bar{B}\epsilon[0,1]$, $\underline{f'}(\varsigma,\bar{B}) \leq \overline{f'}(\varsigma,\bar{B})$ for both triangular and trapezoidal neutrosophic number.

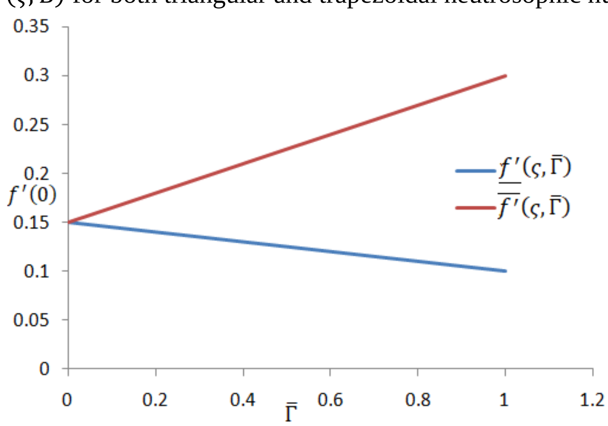

Figure 14(a). Triangular Neutrosophic velocity for $f'(0)$

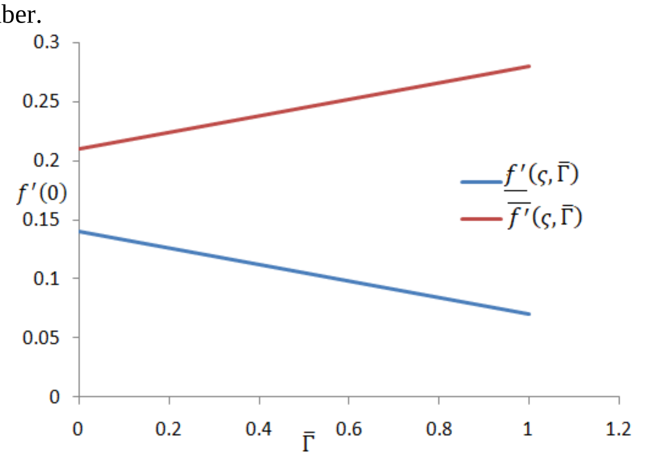

Figure 14(b). Trapezoidal Neutrosophic velocity for $f'(0)$

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From Figures 14(a) and 14(b), we see that, $\underline{f'}(\varsigma,\bar{\Gamma})$ decreases and $\overline{f'}(\varsigma,\bar{\Gamma})$ strictly increases for all $\bar{\Gamma}\epsilon[0,1]$, $\underline{f'}(\varsigma,\bar{\Gamma}) \leq \overline{f'}(\varsigma,\bar{\Gamma})$ for both triangular and trapezoidal neutrosophic number. Hence we can conclude that, the solution of (3.6), $f'(\varsigma,\bar{A},\bar{B},\bar{\Gamma})$ is a strong neutrosophic solution.

Conclusion

In recent years, mathematical modeling that incorporates uncertainty or vagueness has found significant applications across various industries and engineering fields. In this research article, the first-order Differential equation solution is obtained in neutrosophic background. The $(\bar{A},\bar{B},\bar{\Gamma})_{cut}$ neutrosophic set and the strong and weak solutions of neutrosophic DE concepts are also applied for the DE with a $TrapNnumber$ and a linear generalized $TrapNnumber$ as initial conditions. The problem is solves using BVP4C approach. The key findings from the above study are

- The velocity profile upsurges with increase in wedge angle parameter and Magnetic parameter.
- The neutrosophic concept is applied to solve the boundary layer equation whose solution follows the conditions of strong neutrosophic solution.

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