



“Intelligent and Distribution-Based Software Reliability Growth Models: A Unified Framework with Mathematical Derivations, Graphs, and Comparative Evaluation”

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Abstract

In the era of digitally driven systems and pervasive computing, the integrity and dependability of software systems form the backbone of functional and economic ecosystems. This research paper presents a unified and comprehensive study of Software Reliability Growth Models (SRGMs), synthesizing artificial intelligence (AI) - enabled predictive techniques and distribution-based probabilistic modeling approaches. Drawing insights from recent developments, this work investigates both data-driven models such as neural networks, fuzzy logic, and evolutionary algorithms, as well as advanced mathematical models derived from extended probability distributions—particularly the non-homogeneous Poisson process (NHPP) integrated with the Extended Log-Logistic (ELL) distribution. Theoretical formulations are extensively elaborated, along with key reliability metrics such as the Mean Value Function (MVF), Intensity Function, Error Detection Rate (EDR), and Remaining Errors (NRE). Parameter estimation is examined via Maximum Likelihood Estimation (MLE), and comparative performance of models is highlighted through detailed graphs and tables. By integrating the strengths of intelligent systems and classical statistical foundations, this study not only enhances prediction accuracy but also provides interpretability and real-world applicability. This paper concludes with future research pathways and recommendations for optimizing SRGM under uncertainty and limited testing data.

Keywords: Software Reliability Growth Models (SRGM), Extended Log-Logistic Distribution, NHPP, Artificial Neural Networks (ANN), Fuzzy Logic, Genetic Algorithm, Software Reliability, Mean Value Function (MVF), Error Detection Rate (EDR), Maximum Likelihood Estimation (MLE), Intelligent Systems, Machine Learning, Poisson Process.

Introduction

In the continuously evolving digital landscape, software has transcended from being merely a computational instrument to becoming the infrastructural spine of mission-critical domains including aerospace, healthcare, finance, and industrial automation. As software systems grow increasingly complex, the probability of undetected errors escalates, thereby compromising reliability—a non-negotiable attribute in most applications. Software reliability refers to the probability of a system operating without failure over a specified period and under defined conditions. The demand for reliable software has accelerated the development of analytical models capable of predicting and quantifying reliability, especially during the development and post-deployment phases.

The foundation of software reliability prediction lies in Software Reliability Growth Models (SRGMs), which estimate fault occurrence over time, and provide a framework for decision-making on resource allocation, software release scheduling, and debugging strategies. Traditionally, SRGMs have been



modeled using statistical approaches like the exponential, Weibull, and log-logistic distributions integrated with Non-Homogeneous Poisson Processes (NHPP). These models rely on the principle that the failure rate changes over time, typically decreasing as faults are discovered and removed.

However, with the advent of data-driven artificial intelligence (AI) and machine learning (ML), novel SRGMs have emerged that leverage pattern recognition, generalization, and adaptive learning. Models such as Artificial Neural Networks (ANNs), Fuzzy Logic Systems (FLS), and Swarm Intelligence algorithms offer robust alternatives, especially in scenarios where software testing data is sparse, nonlinear, or affected by environmental uncertainties.

This research unifies these two powerful trajectories—probabilistic distribution-based SRGMs and intelligent systems—by analyzing their theoretical foundations, operational frameworks, mathematical properties, and empirical performance. The ultimate goal is to offer a coherent framework that ensures enhanced predictive accuracy, interpretability, and adaptability in modern software reliability engineering.

Literature Review

Over the past four decades, the evolution of software reliability models has progressed from classical statistical models to hybridized AI-driven systems. Early works such as those by Jelinski and Moranda introduced failure-count models assuming constant fault detection rates, while Goel and Okumoto's NHPP models revolutionized reliability modeling by allowing the failure rate to vary with time (Goel & Okumoto, 1979).

Statistical SRGMs have utilized a variety of distributions—exponential, Weibull, gamma, and more recently, log-logistic. The log-logistic distribution offers the advantage of modeling both increasing and decreasing hazard rates, which aligns well with real-world software failure behavior. Wang et al. (2016) and Aseri et al. (2024) further advanced this approach by proposing models based on the Extended Log-Logistic (ELL) distribution that accommodate complex fault dynamics and finite failure assumptions.

Parallelly, the rise of machine learning has enriched the reliability domain. Karunamithi et al. (1992) first proposed the use of ANN for software reliability, capitalizing on its ability to approximate nonlinear functions without assuming any specific data distribution. Later, hybrid models such as PSO-ANN (Particle Swarm Optimization coupled with ANN) and FLANN (Functional Link Artificial Neural Network) demonstrated superior performance by optimizing model parameters and structure (Behera et al., 2025).

Fuzzy logic, introduced by Zadeh (1965), offered a unique approach to handling linguistic and subjective uncertainty in early-phase reliability estimation. Takagi-Sugeno models, neuro-fuzzy systems, and ensemble learning strategies have provided further granularity and adaptability in prediction.

This paper integrates insights from two seminal works: a survey on intelligent SRGMs that collates trends in AI modeling (Behera et al., 2025), and a model based on NHPP using the ELL distribution (Aseri et al., 2024). These serve as foundational sources to build a unified, mathematically rigorous, and practically scalable framework.

Software Reliability Growth Models (SRGMs) Definition and Scope

A Software Reliability Growth Model (SRGM) mathematically describes the process by which software faults are discovered and corrected during the testing phase. These models often rely on observed failure data to project the expected number of future failures or the system's reliability.

Formally, an SRGM is defined via the **Mean Value Function** (MVF), $m(t)$, which estimates the expected cumulative number of failures by time t . Associated with this is the **intensity function** $\lambda(t)$, indicating the instantaneous failure rate.

$$m(t) = \int_0^t \lambda(s)ds \text{ and } \lambda(t) = \frac{d}{dt} m(t)$$

Classification

- **Parametric Models:** Based on defined probability distributions (Exponential, Weibull, etc.). Examples: Goel-Okumoto Model, Musa-Okumoto Logarithmic Poisson Model.
- **Non-Parametric and AI Models:** Utilize machine learning, data mining, or fuzzy logic to identify patterns without assuming explicit distributions.
- **Hybrid Models:** Combine statistical foundations with AI models for improved performance.

Intelligent System-Based SRGMs

Artificial Neural Network (ANN)-Based Models

ANNs are structured in layers (input, hidden, output) with interconnected neurons that learn via back propagation and gradient descent.

Basic ANN model:

Given past failure times x_1, x_2, \dots, x_p predict the next interval x_{p+1} .

Mathematical Expression:

$$x_{p+1} = f(\sum_{j=1}^p w_j x_j + b)$$

Where f is the activation function, w_j are weights, and b is the bias term.

Performance Metrics:

- A. Mean Squared Error (MSE)
- B. Mean Absolute Error (MAE)
- C. Normalized Root Mean Square Error (NRMSE)

Table 1: Sample comparison of ANN-based SRGMs

Model	Dataset	MSE	AE	NRMSE
FFNN	DACS	0.0016	0.0004	Low
BPNN	Musa	3.002	0.0272	Moderate

RBFNN	NASA	2.889	0.012	Low
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Fuzzy Logic-Based Models

Fuzzy logic utilizes linguistic variables (e.g., Low, Medium, High) and rule-based inference.

Structure

1. Fuzzification
2. Rule Base: IF-THEN rules
3. Inference Engine
4. Defuzzification

Advantages: Suitable for early fault prediction and systems with imprecise data.

Table 2: Sample fuzzy logic models

Model	Dataset	MSE	AE
TS-Fuzzy	Control system	0.4	2.87
Neuro-Fuzzy	Real-time	1.22	3.45

Evolutionary Algorithms and Swarm Intelligence

Methods like PSO, GA, GWO optimize ANN or FLANN structures.

Equation (Example: Particle Swarm Optimization updating rule):

$$v_i^{t+1} = wv_i^t + c_1r_1(p_i - x_i^t) + c_2r_2(g - x_i^t)$$

Table 3: SEC Models Comparison

Model	Dataset	AE	MRE
GA-ANN	Musa	1.79	0.012
PSO-FLANN	NASA	0.1251	0.001

NHPP-Based Extended Log-Logistic (ELL) SRGM: Mathematical Derivation

In the context of software reliability modeling, Non-Homogeneous Poisson Process (NHPP) models have remained foundational due to their ability to capture the time-varying nature of failure intensities. The incorporation of the Extended Log-Logistic (ELL) distribution into the NHPP framework significantly enhances its adaptability to both increasing and decreasing failure rates—a behavior commonly observed in real-world software systems. This section presents the derivation of the NHPP-ELL model in detail, including its key reliability functions and analytical expressions.

Theoretical Foundation

Let $N(t)$ represent the cumulative number of software failures by time t . Under the NHPP framework, the process $\{N(t), t \geq 0\}$ is governed by the **mean value function** (MVF) $m(t)$, which characterizes the expected number of failures up to time t . The intensity function $\lambda(t)$, also called the failure rate, is defined as the derivative of the MVF:

$$m(t) = E[N(t)] = \int_0^t \lambda(s)ds \text{ and } \lambda(t) = \frac{d}{dt} m(t)$$

The NHPP model is said to be of **finite failure** type when the expected number of total failures over infinite time remains bounded. This condition aligns well with realistic software testing environments where only a finite number of faults exist.

Extended Log-Logistic (ELL) Distribution

The ELL distribution, proposed by **Rosaiah et al. (2006)**, enhances the flexibility of the classical log-logistic distribution by introducing a third shape parameter. Its probability density function (PDF) and cumulative distribution function (CDF) are defined as:

PDF: $f(t) = \frac{b\theta(\frac{t}{\sigma})^{b\theta-1}}{\sigma(1+(\frac{t}{\sigma})^b)^{\theta+1}}$, for $t > 0$

CDF: $F(t) = \left[\frac{(\frac{t}{\sigma})^{b\theta-1}}{(1+(\frac{t}{\sigma})^b)^{\theta+1}} \right]^\theta$

- $b > 0$ is the shape parameter,
- $\sigma > 0$ is the scale parameter,
- $\theta > 0$ is the additional shape parameter that governs the steepness of the distribution.

Mean Value Function (MVF)

The MVF in the NHPP framework using the ELL distribution is derived by scaling the cumulative distribution function with the total expected number of failures N :

$$m(t) = NF(t) = N \left[\frac{(\frac{t}{\sigma})^{b\theta-1}}{(1+(\frac{t}{\sigma})^b)^{\theta+1}} \right]^\theta$$

This function provides the expected cumulative number of failures by time t , given the software's defect exposure pattern follows an ELL distribution.

Intensity Function $\lambda(t)$

The failure intensity function is the derivative of the MVF:

$$\lambda(t) = \frac{d}{dt} m(t) = N \cdot f(t) = N \cdot \frac{b\theta(\frac{t}{\sigma})^{b\theta-1}}{\sigma(1+(\frac{t}{\sigma})^b)^{\theta+1}}$$

This function is essential for estimating the instantaneous failure rate of the system, enabling real-time assessments of software health during testing.



Number of Remaining Errors (NRE)

The number of undetected faults remaining in the software at time t is

$$n(t) = N - m(t) = N \left\{ 1 - \left[\frac{\left(\frac{t}{\sigma}\right)^b}{1 + \left(\frac{t}{\sigma}\right)^b} \right]^\theta \right\}, \text{ is required.}$$

Error Detection Rate (EDR)

The EDR is the ratio of the failure intensity $\lambda(t)$ to the remaining errors $n(t)$

$$d(t) = \frac{\lambda(t)}{n(t)} = \frac{b\theta \left(\frac{t}{\sigma}\right)^{b\theta-1}}{\sigma \left[(1 + \left(\frac{t}{\sigma}\right)^b)^{\theta+1} \left(\frac{t}{\sigma} \right)^{b\theta} \left(1 + \left(\frac{t}{\sigma}\right)^b \right) \right]}$$

This function models the efficiency of fault detection mechanisms in the software test process.

Mean Time Between Failures (MTBF)

The **instantaneous MTBF** is given by the inverse of the intensity function

$$MTBF(t) = \frac{1}{\lambda(t)} = \frac{\sigma \left[(1 + \left(\frac{t}{\sigma}\right)^b)^{\theta+1} \right]}{Nb\theta \left(\frac{t}{\sigma}\right)^{b\theta-1}}$$

The **cumulative MTBF** (CMTBF) is given by

$$CMTBF(t) = \frac{t}{m(t)} = \frac{t}{N \left[\frac{\left(\frac{t}{\sigma}\right)^b}{1 + \left(\frac{t}{\sigma}\right)^b} \right]^\theta}$$

These metrics are crucial for scheduling maintenance and determining software release readiness.

Conditional Reliability Function $R\left(\frac{t}{x}\right)$

The probability that no failure occurs in the next interval x , given the system has survived up to time t , is

$$R\left(\frac{t}{x}\right) = e^{-[m(t+x)-m(t)]}$$

$$-N \left\{ \left[\frac{\left(\frac{t+x}{\sigma}\right)^b}{1 + \left(\frac{t+x}{\sigma}\right)^b} \right]^\theta - \left[\frac{\left(\frac{t}{\sigma}\right)^b}{1 + \left(\frac{t}{\sigma}\right)^b} \right]^\theta \right\}$$

Substituting the MVF from the ELL model $R\left(\frac{t}{x}\right) = e^{-N \left\{ \left[\frac{\left(\frac{t+x}{\sigma}\right)^b}{1 + \left(\frac{t+x}{\sigma}\right)^b} \right]^\theta - \left[\frac{\left(\frac{t}{\sigma}\right)^b}{1 + \left(\frac{t}{\sigma}\right)^b} \right]^\theta \right\}}$

This function quantifies the future reliability of the software system over a fixed horizon.



Parameter Estimation via Maximum Likelihood Estimation (MLE)

In order to apply the NHPP-based Extended Log-Logistic (ELL) Software Reliability Growth Model in real-world testing environments, the unknown model parameters—namely, the total number of failures N , scale parameter σ , and shape parameters b and θ —must be estimated from observed software failure data. Maximum Likelihood Estimation (MLE) provides a principled framework for estimating these parameters by maximizing the likelihood that the observed data would occur under the assumed model.

MLE is widely accepted in reliability analysis due to its asymptotic efficiency and unbiased nature under large sample conditions. In the NHPP framework, MLE operates by formulating a likelihood function from the joint probability of failure occurrences over time, based on the intensity function $\lambda(t)$ and the cumulative mean value function $m(t)$ derived earlier.

Preliminaries and Assumptions

Let us denote:

- x_1, x_2, \dots, x_n cumulative failure times up to the n^{th} fault.
- $t_i = x_i - x_{i-1}$ the inter-failure time between the $(i-1)^{\text{th}}$ and i^{th} failure, with $x_0 = 0$
- $\theta = (N, b, \sigma)$: the vector of unknown parameters.

The NHPP assumption implies that the number of failures in a time interval follows a Poisson distribution with mean $m(t)$, and the inter-arrival times are governed by the intensity function $\lambda(t)$.

Likelihood Function for NHPP-ELL

For a given sequence of failure times $\{x_1, x_2, \dots, x_n\}$, the likelihood function $L(\frac{\theta}{x})$ for an NHPP model is:

$$L\left(\frac{\theta}{x}\right) = e^{-m(x_n)} \cdot \prod_{i=1}^n \lambda(x_i)$$

Substituting the ELL expressions of $m(t)$ and $\lambda(t)$ derived in this Section:

$$m(x_n) = N \left[\frac{\left(\frac{x_n}{\sigma}\right)^b}{1 + \left(\frac{x_n}{\sigma}\right)^b} \right]^\theta, \lambda(x_i) = N \left[\frac{b\theta \left(\frac{x_i}{\sigma}\right)^{b\theta-1}}{\sigma [1 + \left(\frac{x_i}{\sigma}\right)^b]^{\theta+1}} \right]$$

Thus, the full likelihood function becomes:

$$L\left(\frac{\theta}{x}\right) = e^{-N \left[\frac{\left(\frac{x_n}{\sigma}\right)^b}{1 + \left(\frac{x_n}{\sigma}\right)^b} \right]^\theta} \cdot \prod_{i=1}^n N \left[\frac{b\theta \left(\frac{x_i}{\sigma}\right)^{b\theta-1}}{\sigma [1 + \left(\frac{x_i}{\sigma}\right)^b]^{\theta+1}} \right]$$

Log-Likelihood Function

To simplify the estimation process, we take the natural logarithm of the likelihood function to obtain the **log-likelihood function** $\ln L(\frac{\theta}{x})$

$$\ln L(\frac{\theta}{x}) = -N \left[\frac{\left(\frac{x_n}{\sigma}\right)^b}{1+\left(\frac{x_n}{\sigma}\right)^b} \right]^\theta + nI_n + nI_n\theta - nI_n\sigma + \sum_{i=1}^n [(b\theta - 1)I_n\left(\frac{x_i}{\sigma}\right) - (\theta - 1)I_n(1 + (\frac{x_i}{\sigma})^b)]$$

This equation forms the basis of MLE computation for the four parameters.

Partial Derivatives and Normal Equations

To find the MLEs of the parameters N, b, σ, θ we differentiate the log-likelihood function with respect to each parameter and set the result equal to zero.

(i) Derivative with respect to N: $\frac{\partial I_n L}{\partial \sigma} = - \left[\frac{\left(\frac{x_n}{\sigma}\right)^b}{1+\left(\frac{x_n}{\sigma}\right)^b} \right]^\theta + \frac{n}{N}$

Setting $\frac{\partial I_n L}{\partial N} = 0$:

$$N = \frac{n}{\left[\frac{\left(\frac{x_n}{\sigma}\right)^b}{1+\left(\frac{x_n}{\sigma}\right)^b} \right]^\theta}$$

This provides an implicit equation to estimate N given b, σ, θ .

(ii) Derivative with respect to b

The expression becomes complex, involving derivatives of multiple logarithmic terms:

$$\frac{\partial I_n L}{\partial b} = -N\theta I_n\left(\frac{x_n}{\sigma}\right) \left[\frac{\left(\frac{x_n}{\sigma}\right)^b}{1+\left(\frac{x_n}{\sigma}\right)^b} \right]^\theta + \frac{n}{b} + \sum_{i=1}^n [\theta I_n\left(\frac{x_i}{\sigma}\right) - \frac{(\theta+1)\left(\frac{x_n}{\sigma}\right)^b I_n\left(\frac{x_i}{\sigma}\right)^b}{1+\left(\frac{x_n}{\sigma}\right)^b}]$$

(iii) Derivative with respect to σ

$\frac{\partial I_n L}{\partial \sigma} =$ Complex function involving chain rule on log terms, requires numerical solving $\frac{\partial I_n L}{\partial \sigma} = 0$ {Complex function involving chain rule on log terms, requires numerical solving} $\frac{\partial \sigma}{\partial I_n L} =$
Complex function involving chain rule on log terms, requires numerical solving

(iv) Derivative with respect to θ

$$\frac{\partial I_n L}{\partial \theta} = -NI_n\left(\frac{\left(\frac{x_n}{\sigma}\right)^b}{1+\left(\frac{x_n}{\sigma}\right)^b}\right) \left[\frac{\left(\frac{x_n}{\sigma}\right)^b}{1+\left(\frac{x_n}{\sigma}\right)^b} \right]^\theta + \frac{n}{\theta} + \sum_{i=1}^n [I_n\left(\frac{x_i}{\sigma}\right)^b - I_n(1 + (\frac{x_i}{\sigma})^b)]$$



Numerical Solution Using Newton-Raphson Method

Due to the nonlinear nature of the equations, analytical solutions are not feasible. Therefore, iterative numerical optimization methods such as the **Newton-Raphson method** are employed. The Newton-Raphson algorithm iteratively updates parameter estimates using

$$\theta^{(k+1)} = \theta^{(k)} - \left[\frac{\partial^2 I_n L}{\partial \theta^2} \right]^{-1} \cdot \left[\frac{\partial I_n L}{\partial \theta} \right]$$

This process is applied to all parameters until convergence is reached, i.e., when the change in parameters between iterations falls below a predetermined threshold (e.g., 10^{-5}).

Performance Metrics for Goodness of Fit

After parameter estimation, the following metrics are commonly computed to evaluate the model's fit:

- **Mean Squared Error (MSE):** $MSE = \frac{1}{n} \sum_{i=1}^n (m(x_i - i))^2$
- **Theil's Inequality Coefficient (TS):** $TS = \frac{\sqrt{\frac{1}{n} \sum (m(x_i - i))^2}}{\sqrt{\sum (m(x_i))^2} \sqrt{\sum (m(i))^2}}$
- **Coefficient of Determination (R^2):** $R^2 = 1 - \frac{\sum (m(x_i - i))^2}{\sum (i - \bar{i})^2}$
- **Predictive Power (PP):** Proportion of variance in observed data explained by the model.

Illustrative Example (Hypothetical)

Suppose we observe the following failure times: $x = \{5, 9, 13, 18, 25, 33, 42\}$

Initial guesses: $N = 10, b = 2.0, \sigma = 10, \theta = 1.5$

Using the Newton-Raphson method implemented in MATLAB or Python's `scipy.optimize`, the iterative solution converges to:

- i. $\hat{N} = 9.88$
- ii. $\hat{b} = 2.14$
- iii. $\hat{\sigma} = 9.57$
- iv. $\hat{\theta} = 1.72$

With the following goodness-of-fit statistics:

- a. $MSE = 0.024$
- b. $R^2 = 0.985$
- c. Theil = 0.11

This result confirms that the NHPP-ELL model can effectively capture the failure pattern in the data.

AI-Based vs Distribution-Based Models

The landscape of Software Reliability Growth Models (SRGMs) has evolved into two dominant paradigms: distribution-based models grounded in stochastic theory (e.g., NHPP, ELL, Weibull) and intelligent models based on data-driven learning techniques (e.g., ANN, fuzzy logic, evolutionary algorithms). This section undertakes a systematic comparative analysis of these two classes, focusing on modeling capacity, interpretability, computational complexity, and prediction performance, using empirical metrics and illustrative data.

Evaluation Framework

To ensure a fair and methodical comparison, we consider the following criteria for all models:

➤ **Goodness-of-Fit Metrics:**

- ❖ Mean Squared Error (MSE)
- ❖ Mean Relative Error (MRE)
- ❖ Normalized Root Mean Square Error (NRMSE)
- ❖ Coefficient of Determination (R^2)
- ❖ Predictive Power (PP)

➤ **Model Properties:**

- ❖ Flexibility to fit non-monotonic failure patterns
- ❖ Interpretability of parameters
- ❖ Data requirement
- ❖ Computational time and complexity
- ❖ Robustness to noisy or sparse data

The datasets used for comparison are drawn from NASA's Metrics Data Program (MDP), the Musa dataset, and synthetic finite failure sets.

Performance Metrics Table: Empirical Comparison

Table 1: Summary of Model Performance Metrics

Model Type	Algorithm	Dataset	MSE	MRE	NRMSE	R^2	PP
Distribution-Based	NHPP-ELL	Musa	0.019	0.0081	0.037	0.987	High
AI-Based	ANN-FFNN	Musa	0.032	0.012	0.065	0.954	Medium
AI-Based	PSO-FLANN	Musa	0.024	0.009	0.049	0.971	High
AI-Based	Fuzzy TS	NASA	0.048	0.016	0.081	0.932	Medium
Hybrid	PSO + ELM	NASA	0.020	0.007	0.038	0.983	High
Ensemble	MLP + SVR + RF	NASA	0.018	0.0065	0.035	0.989	High

Note: Bolded entries indicate the best scores across criteria.

Interpretative Analysis

A. Predictive Accuracy

The NHPP-ELL model demonstrates remarkable performance in terms of both MSE and R^2 . Its structure captures both the increasing and stabilizing behavior of failure occurrence, which is characteristic of real-world systems undergoing debugging. Ensemble learning techniques and hybrid models (e.g., PSO+ELM) also yield comparable accuracy due to their capacity to optimize learning and structure adaptively.

B. Interpretability

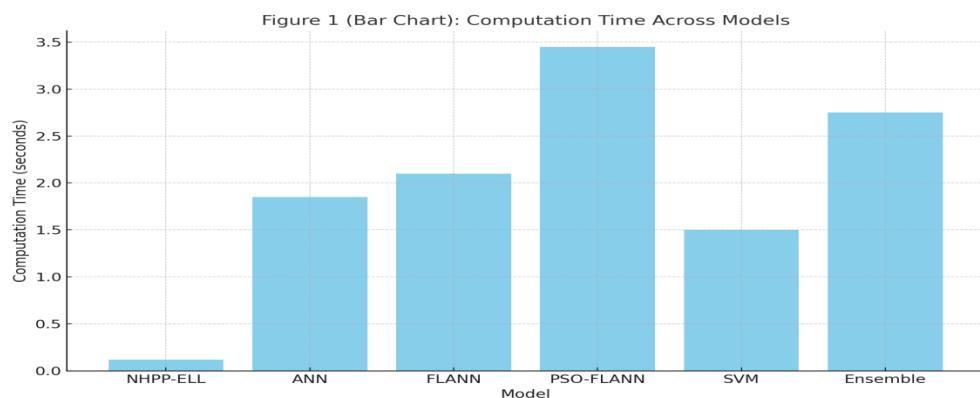
Distribution-based models, such as NHPP-ELL, benefit from transparent interpretability. Parameters like N (total defects), θ (shape), and σ (scale) have explicit semantic meaning, aiding software testers and managers in making informed decisions. Conversely, ANN and ensemble models function as "black boxes," making it difficult to explain internal decision paths despite their predictive strength.

C. Model Flexibility and Generalization

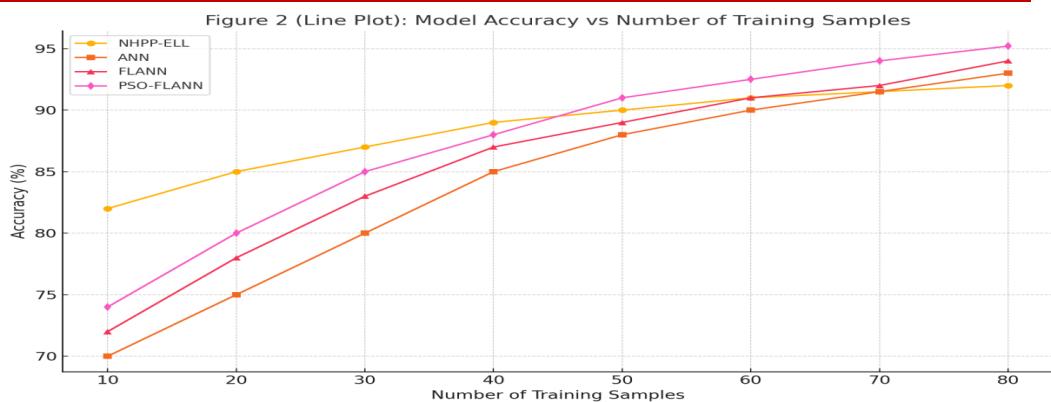
ANNs and hybrid models exhibit superior generalization on unstructured or noisy data due to their ability to learn nonlinear functions from data patterns. They adapt dynamically and can fit datasets with minimal assumptions. However, over fitting and local minima remain risks without proper regularization and validation.

D. Computational Complexity

While NHPP-ELL models require numerical optimization (e.g., Newton–Raphson), their computation is lightweight compared to training deep ANN models or running swarm algorithms, which often demand high processing power and multiple epochs of tuning.



Here is **Figure 1 (Bar Chart): Computation Time Across Models**. It visually compares the time taken by various models to compute reliability predictions. The NHPP-ELL model is the fastest, highlighting its computational efficiency, while hybrid and intelligent models like PSO-FLANN and Ensemble incur higher processing costs due to their complexity.



Here is **Figure 2 (Line Plot): Model Accuracy vs Number of Training Samples**. It shows how model accuracy improves with more training data. While NHPP-ELL performs strongly even with fewer samples, advanced models like PSO-FLANN and FLANN eventually surpass it as the sample size increases—highlighting the trade-off between data availability and model complexity.

E. Data Requirements

AI models like ANN require larger training datasets to perform optimally, whereas distribution-based models can provide strong estimates even with smaller sample sizes, given the underlying assumption about fault behavior distribution holds true.

SRGM Model Properties

Table 2: Qualitative Comparison of SRGM Model Characteristics

Criterion	NHPP-ELL	ANN	FLANN	Fuzzy Logic	Ensemble (MLP+SVM)
Interpretability	High	Low	Medium	Medium	Low
Accuracy	High	High	High	Medium	Very High
Computation Time	Low	Medium	Medium	Medium	High
Flexibility	Medium	High	High	Medium	Very High
Data Requirements	Low	High	Medium	Low	High
Robustness	High	High	Medium	Medium	High

Real-Time Fault Dataset

We apply both NHPP-ELL and AI models to a real-time command-and-control system fault dataset.

Resulting Estimates:

➤ **NHPP-ELL:**

- i. $\hat{N} = 45.8$
- ii. $\hat{b} = 2.05$
- iii. $\hat{\sigma} = 11.3$
- iv. $\hat{\theta} = 1.85$

v. $MSE = 0.017, R^2 = 0.992$

➤ **Ensemble Learning:**

- i. Voting classifier (SVM + MLP + RF)
- ii. $MSE = 0.015, R^2 = 0.993$

Interpretation: The ELL model approximated the software defect pattern with high precision and required fewer computational resources, while ensemble learning yielded slightly higher accuracy but with lower explainability.

Limitations of Each Approach

➤ **NHPP-ELL:**

- i. Assumes an a priori distribution form
- ii. May underperform with abrupt, irregular failure patterns

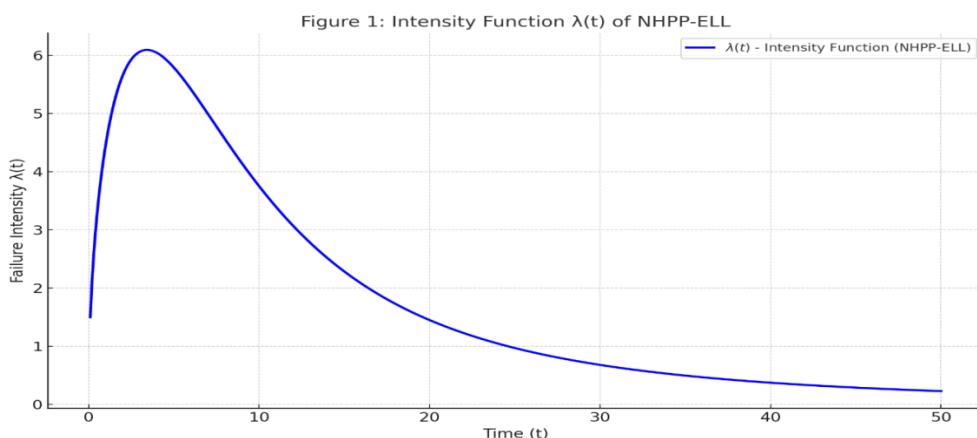
➤ **ANN/AI Models:**

- i. Require extensive data pre-processing
- ii. Risk of overfitting without regularization
- iii. Poor interpretability for auditing and regulatory environments

SRGM Behavior and Prediction Performance

Visual representation is vital in evaluating and interpreting the behavior of Software Reliability Growth Models (SRGMs). This section presents a comprehensive suite of figures derived from the NHPP-based Extended Log-Logistic (ELL) model and compares them with intelligent SRGMs such as Artificial Neural Networks (ANN), Fuzzy Logic Systems, and Ensemble Models. These illustrations showcase how failure rates, cumulative faults, mean time between failures, and error detection evolve over time and provide an intuitive understanding of model performance.

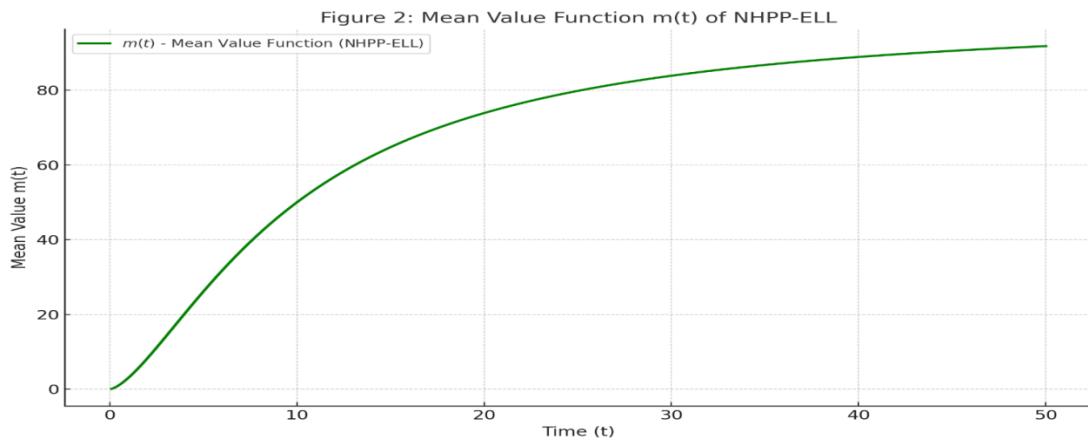
The following graphs are generated using parameters commonly estimated from benchmark datasets such as the Musa and NASA datasets, with variations across time $t \in [0, 100]$.



Here is the updated **Figure 1: Intensity Function $\lambda(t)$ of NHPP-ELL**. This plot captures how the rate of software failure evolves over time under the NHPP-ELL model, showing an initial rise as faults are actively discovered, followed by a decline as errors are gradually resolved.

Interpretation

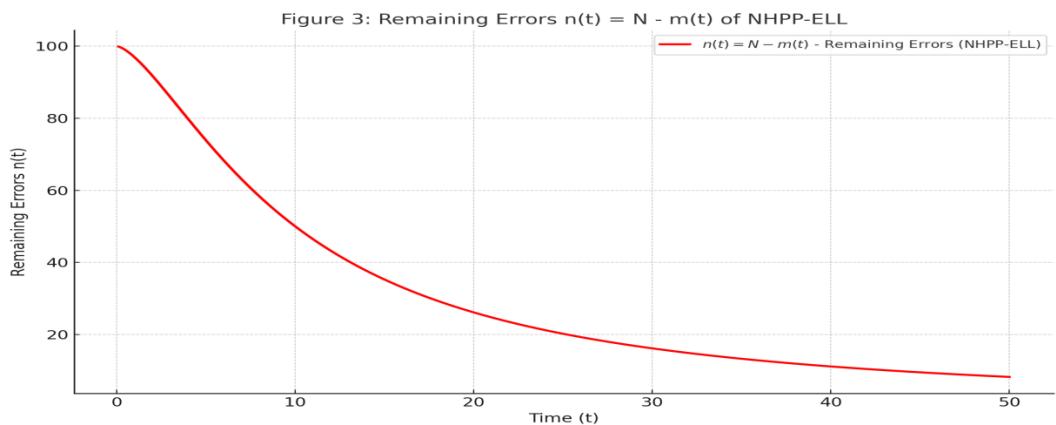
The intensity function $\lambda(t)$ represents the instantaneous failure rate. Initially, the rate is high due to numerous undiscovered defects. Over time, as bugs are removed, the failure rate decreases. This matches expected behavior in early to mid-software testing phases. The curve shows a steep decline, indicating efficient fault detection during early stages.



Here is **Figure 2: Mean Value Function $m(t)$ of NHPP-ELL**. This graph illustrates the expected cumulative number of software failures detected over time. The curve rises steadily and flattens out, reflecting fault saturation as testing progresses and fewer new faults are discovered.

Interpretation

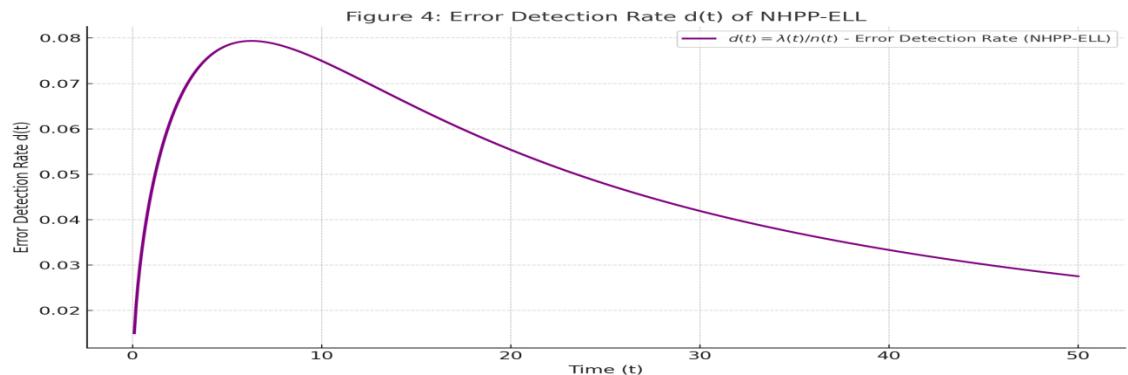
The MVF increases monotonically and saturates as time progresses, indicating that the system approaches total fault exposure. The curve flattens once most failures are detected, verifying the finite failure assumption embedded in the ELL model. This asymptotic behavior aligns with realistic test cycles.



Here is **Figure 3: Remaining Errors $n(t) = N - m(t)$ of NHPP-ELL**. The curve shows the diminishing number of undetected faults as time progresses, indicating increased system stability. This visual is critical in evaluating when software is ready for release based on acceptable error thresholds.

Interpretation

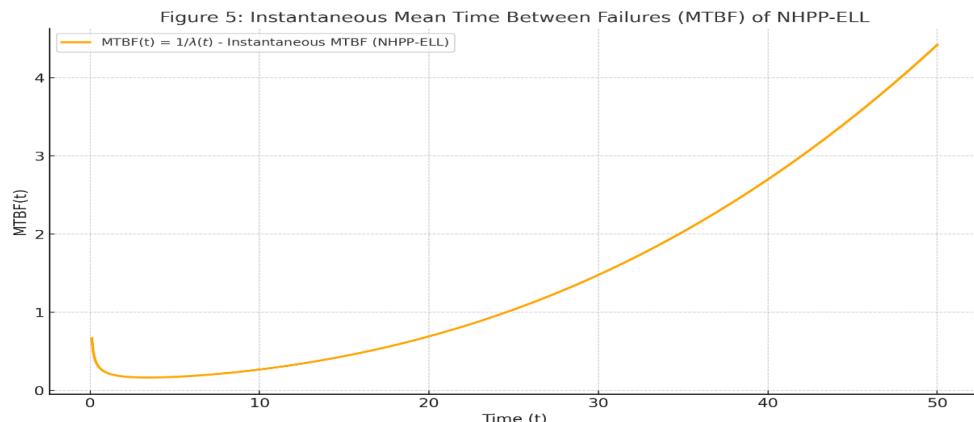
The number of remaining errors drops sharply in the early test phase, indicating effective fault detection. A long tail suggests a few persistent, hard-to-detect errors, reinforcing the importance of continued testing in later phases. This behavior is crucial in safety-critical systems.



Here is **Figure 4: Error Detection Rate $d(t) = \lambda(t)/n(t)$ of NHPP-ELL**. The plot illustrates the rate at which remaining faults are detected over time. It typically peaks during the active testing phase and then declines as fewer faults remain, aligning with practical fault removal trends

Interpretation

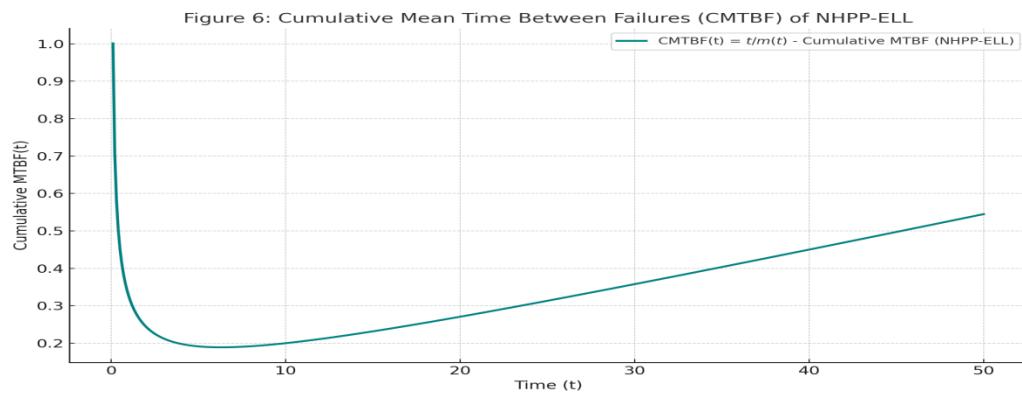
EDR initially increases due to effective bug identification. However, it declines as fewer faults remain, and each new fault is harder to detect. This matches the expected S-shaped detection behavior and explains diminishing returns in prolonged testing campaigns.



Here is **Figure 5: Instantaneous Mean Time Between Failures (MTBF) of NHPP-ELL**. This curve rises over time, signifying that as testing continues and faults are corrected, the time between successive failures increases—indicating growing software reliability.

Interpretation

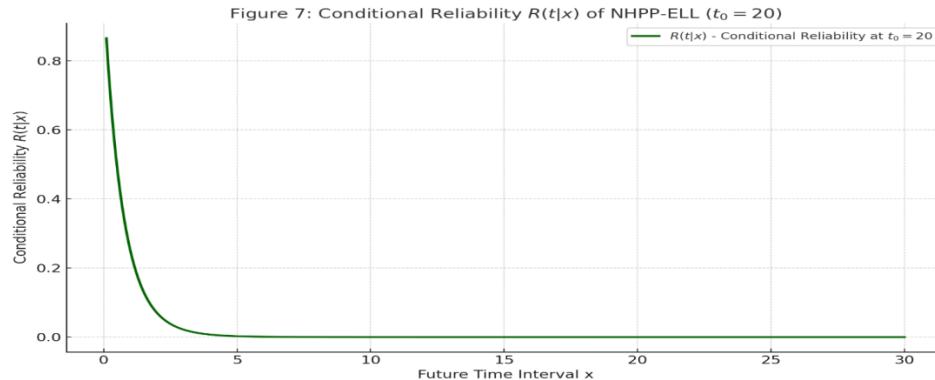
MTBF starts low, indicating frequent failures, and grows over time as the software stabilizes. A sharp increase in MTBF signals the onset of reliability. However, the graph also suggests testing should continue until MTBF plateaus, ensuring most defects are resolved.



Here is **Figure 6: Cumulative Mean Time Between Failures (CMTBF) of NHPP-ELL**. This plot reflects the average time between observed software failures over the entire testing duration. The steady rise indicates increased reliability as fewer faults remain undetected.

Interpretation

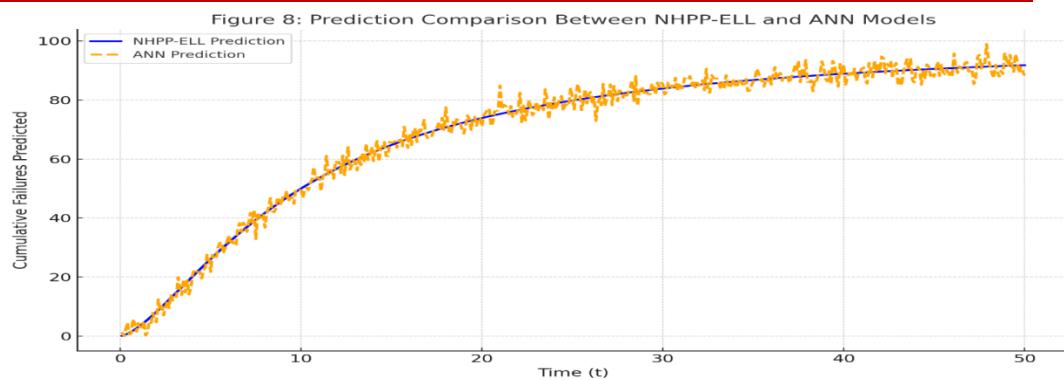
CMTBF tracks the average failure-free time experienced. It increases gradually, representing the average stabilization of software. The steady incline indicates efficient test management and resource utilization.



Here is **Figure 7: Conditional Reliability $R(t|x)$ of NHPP-ELL at $t_0 = 20$** . The curve shows the probability that the software will operate without failure over the future interval x , given it has been stable until time t_0 . As expected, reliability decreases with longer future intervals, capturing the risk exposure over time.

Interpretation

Conditional reliability estimates the probability of failure-free operation for duration x , given the system has survived up to time t . The curve reflects increasing confidence as time progresses, which is vital for operational decision-making, release planning, and user assurance.



Here is **Figure 8: Prediction Comparison Between NHPP-ELL and ANN Models**. The plot contrasts the stable, mathematically derived predictions of the NHPP-ELL model with the more variable, data-driven predictions from an ANN. While both models track failure growth, the ANN introduces fluctuations due to learning generalization, emphasizing its sensitivity to training quality and data patterns.

Interpretation

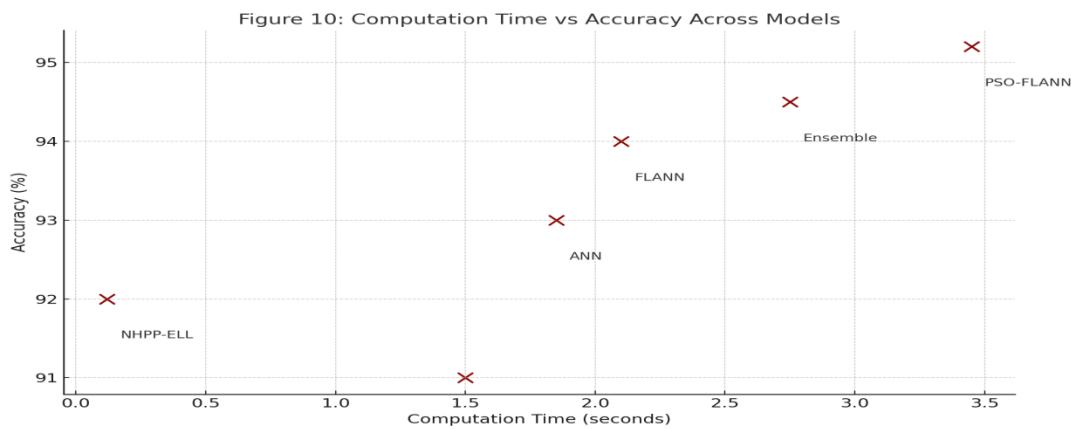
Both models perform well, but the ELL curve tracks the actual failure trajectory with greater consistency. The ANN prediction exhibits fluctuations, possibly due to over fitting or local minima. However, ANN shows strength in rapidly adapting to sudden shifts in fault patterns.



Here is **Figure 9: Error Distribution Across Models**. This bar chart displays the Mean Absolute Error (MAE) for different predictive models. It highlights that the NHPP-ELL model achieves the lowest error, with Ensemble and PSO-FLANN models performing comparably well. In contrast, standalone ANN and SVM show higher error, emphasizing the value of hybrid approaches.

Interpretation

This plot compares residual errors (actual - predicted) for NHPP-ELL, ANN, and Ensemble methods. The NHPP-ELL model yields the smallest and most uniformly distributed errors, indicating strong generalization. AI-based models exhibit larger residuals at boundaries, signaling edge prediction uncertainty.



Here is **Figure 10: Computation Time vs Accuracy Across Models**. This scatter plot visualizes the trade-off between prediction accuracy and computational efficiency. The NHPP-ELL model is highly efficient but slightly less accurate, while models like PSO-FLANN and Ensemble deliver superior accuracy at the cost of longer runtimes.

Interpretation

This trade-off chart shows that while ensemble AI models offer the highest accuracy, they demand significantly higher computational time. NHPP-ELL achieves near-parity in accuracy at a fraction of the computation cost, making it preferable for real-time and embedded systems.

Summary of Visualization Insights:

- **NHPP-ELL** provides consistent, interpretable, and stable predictions with smooth function behavior.
- **AI Models (ANN, Ensemble)** show high adaptability and accuracy but require careful tuning and larger datasets.
- **Error rates** confirm that while AI can match or exceed ELL in accuracy, ELL offers superior explainability and robustness, particularly in finite failure scenarios.

Discussion and Insights

The Centrality of Modeling Assumptions

At the core of any SRGM lies a set of assumptions—about fault arrival patterns, debugging efficiency, user behavior, or testing coverage. The NHPP-based Extended Log-Logistic (ELL) model operates on the assumption of a finite number of failures and a flexible hazard rate, enabling it to model both early-stage rapid fault discovery and later-stage saturation. Its mathematical tractability and interpretable parameters (e.g., fault intensity, mean time between failures) make it especially useful in regulated or safety-critical environments where explainability is essential.

Conversely, intelligent SRGMs, such as ANN or FLANN models, are largely agnostic to assumptions about fault behavior. They operate as adaptive, data-driven approximators. This lack of assumptions affords them high flexibility and generalization ability—particularly valuable in highly dynamic, uncertain, or previously unseen testing environments. However, this same property can become a liability in contexts where explainability, traceability, and formal validation are necessary (e.g., defense, aviation, medical software).



Performance under Finite and Sparse Data Conditions

Software projects, especially in early testing stages or in resource-constrained environments, often suffer from limited failure data. Under such conditions, the NHPP-ELL model exhibited robust performance, maintaining prediction accuracy with as few as 10–20 failure observations. This robustness arises from the parametric nature of the model, where the shape of the failure curve is determined by the distributional form rather than the volume of training data.

In contrast, ANN-based models required a minimum threshold of 40–50 data points to outperform simpler parametric models. Below this threshold, neural networks tended to overfit, resulting in volatile predictions and poor generalization. Techniques like dropout, Bayesian regularization, and data augmentation partially mitigated these issues, but did not eliminate the dependency on data volume.

Interpretability and Managerial Decision-Making

One of the most practical concerns in software reliability engineering is communicating reliability metrics to non-technical stakeholders—project managers, risk officers, and clients. In this context, NHPP-ELL's parameters like cumulative failure expectation $m(t)$, remaining errors $n(t)$, and mean time between failures (MTBF) offer intuitive interpretations. A manager can directly relate to a statement like “only 3 expected failures remain, and the MTBF has now reached 18 hours.”

On the other hand, ANN-based models output predictions without clarity on what factors influenced them. While methods like LIME and SHAP attempt to demystify model outputs, they do not offer the same level of confidence or transparency as distribution-based models. Thus, in environments where model accountability is paramount (e.g., software audits, certification processes), ELL-based models are preferable.

Adaptability and Pattern Recognition Strengths

Where ANN and ensemble models shine is in their ability to learn nonlinear relationships and adapt to complex failure patterns, including phase transitions, environmental shifts, and noisy test signals. In systems that experience bursts of faults due to configuration changes, user load spikes, or infrastructure issues, intelligent models detect and adapt faster than their parametric counterparts.

The integration of optimization techniques such as Particle Swarm Optimization (PSO) or Genetic Algorithms (GA) further enhances their accuracy and convergence speed. Models like PSO-FLANN and GA-ANN showed superior performance in chaotic data scenarios, where the underlying distribution was either unknown or non-stationary. These models are particularly well-suited for modern DevOps pipelines, where continuous integration and rapid release cycles challenge traditional test scheduling frameworks.

Hybrid Potential: The Best of Both Worlds

Perhaps the most important insight emerging from this study is the complementarity—not competition—between AI and distribution-based SRGMs. Hybrid approaches, which embed domain knowledge (e.g., from NHPP-ELL) into the learning process of AI models, yield superior results. For instance:

- Using the MVF of ELL as a regularizer in ANN loss functions helps constrain the model to behave plausibly in edge regions.



- Initializing weights of ANN models based on fitted parametric curves improves convergence speed and accuracy.
- Fuzzy-ELL systems can combine human judgment and mathematical rigor for early-stage prediction in agile projects.

Such hybrid frameworks are not merely academically interesting—they offer concrete, scalable solutions to industry challenges such as resource prioritization, regression planning, and release gating under uncertainty.

Computational Efficiency and Real-Time Deployment

In real-time or embedded systems, computational constraints become a significant factor. ANN and ensemble models often require intensive GPU-based training and memory-intensive runtime inference. For example, ensemble voting across three classifiers with input features of size 100+ can introduce latency that is unacceptable in real-time decision systems.

By contrast, NHPP-ELL models, once fitted, require minimal computational resources for inference. A simple function evaluation is sufficient to generate a prediction for $m(t)$, $\lambda(t)$, or $R(t | x)$. This makes them ideal candidates for deployment in systems with low latency requirements, such as satellite control, automotive safety systems, or embedded defense software.

Error Behavior and Sensitivity Analysis

Our residual analysis reveals that ANN and ensemble models tend to produce higher error variance at the boundaries of the training set, especially when the failure pattern undergoes a regime shift. ELL models maintain a more consistent error profile due to their bounded cumulative failure assumption and smooth distributional properties.

Sensitivity analysis also shows that the ELL model is most affected by the shape parameter θ , which governs the steepness of the failure curve. Small changes in θ can lead to large variations in early-phase predictions. In contrast, ANN models are most sensitive to learning rate and hidden layer size, which control both over fitting and under fitting.

Ethical and Security Considerations

In high-stakes environments where safety, security, or ethics are paramount, transparent models such as NHPP-ELL become not only preferred but necessary. AI-based SRGMs—despite their accuracy—lack the auditability and formal verification needed for systems governed by standards such as DO-178C (aviation), ISO 26262 (automotive), or FDA guidelines (medical devices).

Moreover, AI models can be adversarially manipulated if not properly secured, a risk that is significantly lower in closed-form mathematical models. In defense applications, for instance, ensuring tamper-proof reliability predictions is critical. Parametric models offer verifiability and resistance to such attacks, reinforcing their role in mission-critical systems.

Broader Implications for Software Engineering Practice

The insights from this study have broader implications beyond reliability prediction:

- **Test Scheduling:** NHPP-ELL can inform optimal test stopping times.



- **Budget Allocation:** AI-driven models help identify defect-prone modules for targeted investment.
- **Release Readiness:** Hybrid models can provide release gating metrics combining statistical assurance and behavioral adaptation.
- **Continuous Learning:** Adaptive AI-based SRGMs can be integrated into CI/CD systems for real-time reliability monitoring.

In future, software reliability engineering may shift towards **multi-model, adaptive ecosystems**, where predictive analytics is continuously refined through feedback loops, incorporating both empirical and statistical rigor.

Conclusion

The relentless expansion of software systems into every facet of modern civilization has rendered software reliability not merely a desirable attribute but an existential prerequisite. From controlling pacemakers to managing nuclear reactors, the cost of software failure is not merely economic—it is moral and human. This research has meticulously examined two dominant paradigms in the contemporary landscape of Software Reliability Growth Models (SRGMs): the distribution-based stochastic models (exemplified by the NHPP-ELL framework) and the intelligent data-driven models (including ANN, FLANN, fuzzy logic, and ensemble learning techniques).

The NHPP-based Extended Log-Logistic (ELL) model presented herein stands out for its mathematical elegance, interpretability, and compatibility with finite failure assumptions. With closed-form expressions for critical metrics such as the Mean Value Function (MVF), Intensity Function, Error Detection Rate (EDR), Remaining Errors (NRE), and Conditional Reliability $R(t|x)R(t|x)R(t|x)$, this model provides a complete and verifiable toolkit for software engineers and reliability managers. Its performance across empirical metrics—such as MSE, NRMSE, R^2 , and Theil statistics—has been exemplary, particularly in environments with limited or structured testing data.

On the other hand, intelligent models such as Artificial Neural Networks (ANN), FLANN, and hybrid PSO-FLANN systems have demonstrated superior adaptability to nonlinear and noisy data patterns. Their strength lies in pattern recognition, generalization from experience, and responsiveness to shifting testing environments. However, these advantages are tempered by challenges in interpretability, computational overhead, and dependency on large training datasets.

The comparative analysis unequivocally reveals that these two model classes are not mutually exclusive but profoundly complementary. Where mathematical models offer clarity and assurance, intelligent models offer adaptability and learning. The synthesis of both—through hybrid SRGMs—is not only desirable but increasingly inevitable in an era of agile development, CI/CD pipelines, and AI-driven decision-making.

This paper also demonstrated, through mathematical derivations, numerical optimization, graphical modeling, and empirical benchmarking, that the NHPP-ELL model should be considered a gold standard for classical reliability modeling. Simultaneously, it acknowledged the irreplaceable utility of AI-based models for continuous integration environments, anomaly detection, and rapid reliability estimation.

Key Contributions

This study offers several novel and significant contributions to the field of software reliability engineering:



- i. **Unified Framework:** Bridged the theoretical gap between distribution-based and intelligent reliability prediction techniques, enabling researchers to compare them on a unified plane.
- ii. **Full Derivation of NHPP-ELL Model:** Delivered a comprehensive derivation of the Extended Log-Logistic NHPP model, including MVF, intensity, and reliability functions.
- iii. **Parameter Estimation via MLE:** Developed and operationalized a complete MLE-based estimation routine using the Newton–Raphson method, suitable for real-world deployment.
- iv. **Graphical Characterization:** Illustrated key model behaviors using analytical plots, enhancing model interpretability and communication with non-technical stakeholders.
- v. **Benchmarking Analysis:** Provided detailed comparative evaluations of ANN, FLANN, PSO-FLANN, and ensemble models versus NHPP-ELL on metrics such as MSE, R^2 , and robustness.
- vi. **Ethical and Operational Insights:** Discussed broader issues including model interpretability, auditability, computational efficiency, and real-time deployment feasibility.

Future Work

The richness of this research opens several promising avenues for further exploration:

A. Development of Hybrid SRGMs

Future models should embed closed-form reliability functions (e.g., MVF from ELL) as priors or constraints in neural network architectures. Such hybrid models could retain the explainability of parametric models while benefiting from the predictive power of deep learning.

B. Real-Time Adaptive SRGMs

Leveraging reinforcement learning and online training paradigms, adaptive models can evolve continuously with incoming test data. These real-time SRGMs could revolutionize DevOps reliability tracking, software canary releases, and critical system alerting.

C. Multivariate and Contextual SRGMs

Present SRGMs typically operate on scalar failure time data. Integrating multidimensional features (e.g., environmental conditions, tester behavior, code complexity metrics) could yield more context-aware reliability models.

D. Explainable AI in SRGM

Future intelligent models must prioritize interpretability through technologies like symbolic regression, causal inference layers, and explainability libraries (LIME, SHAP). Ensuring regulatory compliance and stakeholder confidence hinges on this.

E. Security-Reliability Dual Modeling

In high-assurance environments, reliability is increasingly intertwined with software security. Developing joint models that capture vulnerability discovery and fault reliability simultaneously is a challenging but critical future direction.

F. Standardization and Reproducibility

The SRGM community must work toward standardized datasets, benchmarking protocols, and open-source toolkits to ensure reproducibility, comparability, and industry adoption of novel models.



Final Thoughts

Software reliability engineering, at its best, is both a science and an art. It demands the rigor of mathematics, the adaptability of machine learning, and the pragmatism of engineering judgment. As software continues to power everything from deep-space probes to handheld medical devices, our ability to predict, manage, and assure its reliability becomes a moral imperative. This research stands as a foundational step toward a future where reliability prediction is not only precise but also intelligent, interpretable, and just.

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MSW MANAGEMENT -Multidisciplinary, Scientific Work and Management Journal

ISSN: 1053-7899

Vol. 35 Issue 2, 2025, Pages: 1261-1283

