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Performance Analysis of a Two -Unit Cold Standby Redundant System with Priority in Operation, Degradation and Preventive Maintenance

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Abstract
This paper discusses a two-unit cold standby system in which one unit is operational and the other is in cold standby mode, i.e. the standby unit is utilised to replace the operationally failed unit immediately. The concept of inspections, degradation, and priority in operation to the new unit over degraded unit and preventive maintenance (PM) after maximum operation time are used. There are two type of servers expert and ordinary that visit the system anytime when it is required and the servers is capable of performing all types of repairs, inspections, and preventative maintenance (PM). PM is provided to the system when both the new units are available for use. The new unit works with less capacity after its repair by ordinary server and is called degraded unit. As and when a degraded unit fails, it is inspected by expert server to determine that it can be repaired; if this is not possible, it is immediately replaced by a new unit. If any two of the new/degraded units are operational, the system is considered in up-state. All random variables follow arbitrary distribution. The regenerative point technique and semi-Markov process is used to obtained the system measures such as mean time to system failure (MTSF), mean sojourn time, availability, busy period of the servers, predicted quantity of the visits by using the servers and expected number of visits by server. For a particular case, the numerical results are also evaluated for MTSF, availability and profit of the system model.

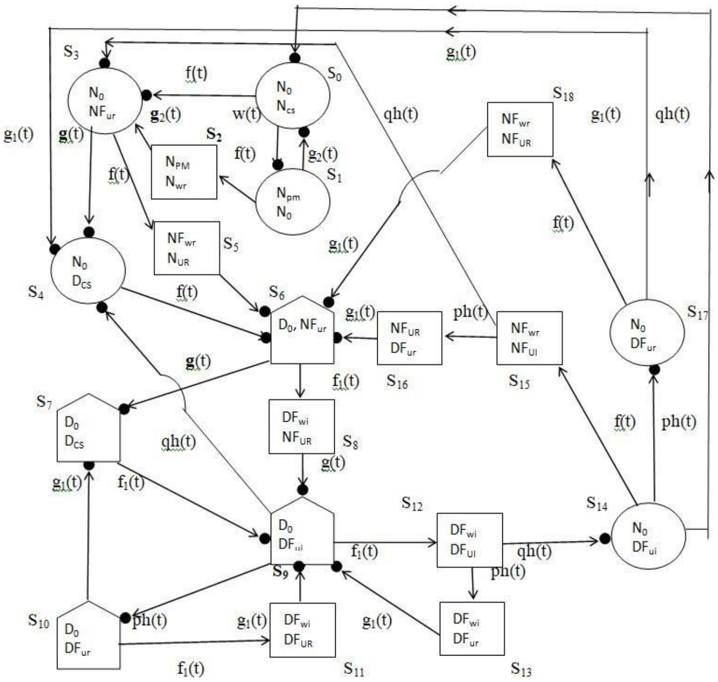
Keywords
Two-unit system, Preventive Maintenance, Maximum Operation Time, Degradation, Inspection and Replacement.

1. Introduction

In our day-to-day life we can encounter many examples in which the quality of the production plays an important role, hence it is necessary to Impose exceptionally high standards on their performance and estimate the reliability of performance for a specified time. When we buy components (systems), we expect them to function properly for a reasonable period of time. Many engineer and researchers try their best to achieve this target, like as Agnihotri and Satsangi (1996) carried out reliability analysis of a two non- identical unit under priority repair and inspection using regenerative point technique. Chan and Asgarpoor (2001) discussed the preventive maintenance with markov process. Grall et al.

(2002) presented a preventive maintenance structure for a gradually deteriorating single-unit system. Malik et al. (2008) performed the reliability and profit evaluation of an operating system with different repair strategy. Kiureghain and Ditlevson (2007) discussed the availability, reliability and downtime of system with repairable components. El said and Sherbeny [2010] studied a cold standby system with two stage repair of the failed unit. Kumar and Kadyan [2012] did the economic analysis of a standby system with degradation and replacement. Wang and Pham (2011) were calculated A multi objective optimization of imperfective preventive maintenance policy with hidden failure rates. In order to increase the system performance with different assumption the system models with preventive maintenance were described by Kumar and goel (2016) discussed a two unit cold standby system by considering the concepts of degradation, inspection, preventive maintenance and priority with general distribution. Ruiz-Castro, Juan Eloy, and Mohammed Dawabsha (2020) studied A multi-state warm standby system with preventive maintenance, loss of units and an indeterminate multiple number of repairpersons. Aggarwal, Chhama, Nitika Ahlawat, and S.C. Malik (2021) discussed Profit Analysis of a Standby Repairable System with Priority to Preventive Maintenance and Rest of Server Between Repairs.

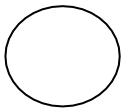
Based on the foregoing observation, In this paper we investigate a two-unit cold standby system with identical units (one is operative and other is in standby), priority in operation is provided to the new unit over degraded unit. When the new unit fails and repaired it becomes degraded unit due to its less working capacity as compare to fresh or new unit. Preventive maintenance is provided to the new and operative unit after its maximum operation time. There are two types of servers that are capable of doing all types of repairs, inspections, and PM of the system as soon as it is needed. The expert server inspects the degraded unit at the site of its failure. If inspection of failed degraded unit reveals that its repair is not useful to the system, it is replaced by new unit. Ordinary server performs PM and repairs of new units, while expert server performs inspection and repair of degraded units when they fail. The system is considered to be in good working condition if any of the new or degraded units are operating. All random variables follow arbitrary distribution. The regenerative point technique and semi-Markov process is used to obtained the system measures such as mean time to system failure (MTSF), mean sojourn time, availability, busy period of the servers, predicted quantity of the visits by using the servers and expected number of visits by server. For a particular case, the numerical results are also evaluated for MTSF, availability and profit of the system model.

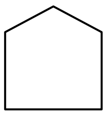


State Transition Diagram of the System

State Description:

Regenerative State : ●

Operative State : 

Degraded State : 

Failed State : 

2. **Assumptions**

Following are the assumptions for the analysis of the Markov model:

1. The system is composed of two similar units, one of which is operative and the other is kept in cold standby. In standby mode, the unit cannot fail.
2. PM will be provided to the system after maximum operative time for increasing the system performance.
3. There are two types of servers-ordinary and expert.
4. After repair, the new unit becomes degraded.
5. Priority in operation is given to the new unit in comparison to the degraded unit.
6. All the random variables follow arbitrary distribution.
7. The server cannot exit the system during inspection and repair of the unit.
8. Switches are perfect.

3. **Notations**

E Set of regenerative states.
No/Do The new/degraded operative unit.
NCs/DCs The new/degraded unit in cold standby.
NF_{ur1}/NF_{UR1} /NF_{wr} New unit is failed and under repair / under continuous repair from previous state/waiting for repair.
DF_{ur2}/DF_{UR2} /DF_{wr} Degraded unit are failed and under repair / under repair continuously from previous state /waiting for repair.
Degraded unit is failed and is under inspection / waiting for Inspection

Degraded unit is under inspection continuously from previous state/waiting for Inspection continuously from previous state.

Probability that repair of degraded unit is possible/not possible.

$f(t)/F(t), f_1(t)/F_1(t)$ p.d.f / c.d.f of failure time for new/degraded unit. $g(t)/G(t), g_1(t)/G_1(t)$ p.d.f / c.d.f of repair time for new/degraded unit. $g_2(t)/G_2(t)$ p.d.f / c.d.f of PM time of new unit.

$h(t)/H(t)$ p.d.f/ c.d.f of inspection time of the degraded unit.

p.d.f/c.d.f of maximum operation time by which unit undergoes for PM.

p.d.f /c.d.f of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* without visiting any other regenerative state in (0,t].

p.d.f/c.d.f of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* visiting state *k,r* once in (0,t].

Probability that system up initially in state $S_i \in E$ is up at time *t* without visiting any other regenerative sate

Probability that server is busy in the state S_i up to time *t* without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states

`(desh) Symbol for derivative of the function.

Symbols for Laplace Stieltjes Transform (L.S.T)/Laplace Transform (L.T).

® /© Symbols for Stieltjes convolution/Laplace convolution.

The following are the possible transition states of the system model:-

$S_0 = (N_0, Ncs)$,	$S_1 = (N_0, N_{pm1})$,	$S_2 = (N_{PM1}, NF_{wr})$,
$S_3 = (N_0, NF_{ur1})$,	$S_4 = (N_0, D_{cs})$,	$S_5 = (NF_{wr}, NF_{UR1})$,
$S_6 = (D_0, NF_{ur1})$,	$S_7 = (D_0, D_{cs})$,	$S_8 = (DF_{wi}, NF_{UR1})$,
$S_9 = (D_0, DF_{ui2})$,	$S_{10} = (D_0, DF_{ur2})$,	$S_{11} = (DF_{wi}, DF_{UR2})$,
$S_{12} = (DF_{wi}, DF_{UI2})$,	$S_{13} = (DF_{wi}, DF_{ur2})$,	$S_{14} = (N_0, DF_{ui2})$,
$S_{15} = (NF_{wr}, DF_{UI2})$,	$S_{16} = (NF_{WR}, DF_{ur2})$,	$S_{17} = (N_0, DF_{ur2})$,
$S_{18} = (NF_{wr}, DF_{UR2})$,		

The states S_0, S_1, S_3, S_4, S_6, S_7, S_9, S_{10}, S_{14} and S_{17} are regenerative states while S_2, S_5, S_8, S_{11}, S_{12}, S_{13}, S_{15}, S_{16} and S_{18} are non - regenerative states . Thus $E = \{S_0, S_1, S_3, S_4, S_6, S_7, S_9, S_{10}, S_{14}, S_{17}\}$.

4. Transition Probability and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non – zero Elements.

$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt$.

$dQ_{0,1}(t) = w(t)F(t)dt$

$dQ_{0,3}(t) = f(t)\overline{W(t)}dt$

$dQ_{1,0}(t) = g_2(t)\overline{F(t)}dt$

$dQ_{1,3.2}(t) = [f(t)G_2(t) © \overline{g_2(t)}]dt$

$dQ_{3,4}(t) = g(t)\overline{F(t)}dt$

$dQ_{3,6.5}(t) = [f(t)G(t) © \overline{g(t)}]dt$

$dQ_{4,6}(t) = f(t)dt$

$dQ_{6,7}(t) = g(t)\overline{F(t)}dt$

$dQ_{6,9.8}(t) = [f_1(t)G(t) © g(t)]dt$

$dQ_{7,9}(t) = f_1(t)dt$

$dQ_{9,10}(t) = [ph(t)\overline{F(t)}]dt$

$dQ_{9,4}(t) = [qh(t)\overline{F_1(t)}]dt$

$dQ_{9,12}(t) = [f_1(t)\overline{H(t)}]dt$

$dQ_{9,14.12}(t) = [f_1(t)H(t) © \overline{qh(t)}]dt$

$dQ_{10,7}(t) = [g_1(t)\overline{F_1(t)}]dt$

$dQ_{10,9.11}(t) = [f_1(t)G_1(t) © \overline{g_1(t)}]dt$

$dQ_{14,0}(t) = [qh(t)\overline{F(t)}]dt$

$dQ_{14,17}(t) = [ph(t)\overline{F(t)}]dt$

$dQ_{14,15}(t) = [f(t)\overline{H(t)}]dt$

$dQ_{14,3.15}(t) = [f(t)H(t) © \overline{qh(t)}]dt$

$dQ_{17,4}(t) = [g_1(t)\overline{F(t)}]dt$

$dQ_{17,6.18}(t) = [f(t)G_1(t) © \overline{g_1(t)}]dt$

For these transition probabilities, it can be verified that:

$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt$ as

$$p_{0,1}(t) + p_{0,3}(t) = p_{1,0}(t) + p_{1,3.2}(t) = p_{4,6}(t) = p_{5,6}(t) = p_{11,9}(t) = p_{7,9}(t) = p_{13,9}(t) = p_{16,6}(t) =$$
$$p_{6,7}(t) + p_{6,9.8}(t) = p_{12,13}(t) + p_{12,14}(t) p_{14,0}(t) + p_{14,17}(t) + p_{14,15}(t) = p_{17,4}(t) + p_{17,18}(t) =$$
$$p_{15,16}(t) + p_{15,3}(t) = 1$$

The mean sojourn time's μ_i in state S_i are given by:

$$\mu_0 = \int_0^\infty F(t)W(t)dt = \frac{1}{\lambda^k + w^k} \qquad \mu_1 = \int_0^\infty G_2(t)F(t)dt = \frac{1}{\lambda^k + r_2^k}$$

$$\mu_3 = \int_0^\infty G(t)F(t)dt = \frac{1}{r^k + \lambda^k} \qquad \mu_4 = \int_0^\infty F(t)dt = \frac{1}{\lambda^k}$$

$$\mu_6 = \int_0^\infty G(t)F_1(t)dt = \frac{1}{r^k + \lambda_1^k} \qquad \mu_7 = \int_0^\infty F_1(t)dt = \frac{1}{\lambda_1^k}$$

$$\mu_9 = \int_0^\infty H(t)F_1(t)dt = \frac{1}{\theta^k + \lambda_1^k} \qquad \mu_{10} = \int_0^\infty G_1(t)F_1(t)dt = \frac{1}{\lambda_1^k + r_1^k}$$

$$\mu_{14} = \int_0^\infty F(t)H(t)dt = \frac{1}{\theta^k + \lambda^k} \qquad \mu_{17} = \int_0^\infty G_1(t)F(t)dt = \frac{1}{r^k + \lambda_1^k}$$

5. Mean Time to System Failure

Suppose $\phi_i(t)$ represents the initial passage time to a failed state from regenerative state i. For $\phi_i(t)$ the recursive relations are represented as below where failed state are considered as absorbing states

$$\phi_i(t) = \sum_j Q_{i,j}(t) \circledR \phi_j(t) + \sum_k Q_{i,k}(t) \qquad \ldots(1)$$

Where j represents an operative regenerative state where the specified regenerative state i can be transited as well as k represents a failed state where the state i can directly transit.

Considering relations (1) and taking L.S.T. as well as explaining for $\phi_0(s)$
Utilizing this, we get

$$R^*(s) = (1 - \phi_0(s)) / s \qquad \ldots(2)$$

For obtaining the reliability $R(t)$ taking L.S.T. of equation (2), the following represents the MTSF as:

$$\text{MTSF } (T_1) = \frac{N}{D} \qquad \ldots(3)$$

where

$$N = [\mu_0 + p_{0,1}(t)\mu_1][1 - p_{4,6}(t)p_{6,7}(t)p_{7,9}(t)p_{9,4}(t) - p_{9,10}(t)p_{10,7}(t)p_{7,9}(t)] + p_{0,3}(t)\mu_3 +$$
$$p_{0,3}(t)p_{3,4}(t)[\mu_4 + p_{4,6}(t)\mu_6 + p_{4,6}(t)p_{6,7}(t)\mu_7] + p_{0,3}(t)p_{3,4}(t)p_{6,7}(t)[\mu_9 + p_{9,10}(t)\mu_{10}]$$

$$D = [1 - p_{0,1}(t)p_{1,0}(t)][1 - p_{4,6}(t)p_{6,7}(t)p_{7,9}(t)p_{9,4}(t) - p_{9,10}(t)p_{10,7}(t)p_{7,9}(t)]$$

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6. Availability Analysis

Suppose A_i *(t)* represents the probabilities that at instant *t,* the system is in up - state provided that at *t = 0*, regenerative state *i* is entered by the system. For A_i *(t)* the recursive relation is as follows:

$$A_i(t) = M_i(t) + \sum q_{i,j}^{(n)}(t) © A_j(t) \qquad \ldots(4)$$

Where *j* represents a regenerative state where through by $n \geq 1$ transitions the transfer is made regenerative state *i*.

$M_0(t) = \overline{F}(t)\overline{W}(t)$

$M_1(t) = \overline{F}(t)\overline{G_2}(t)$

$M_3(t) = \overline{F}(t)\overline{G}(t)$

$M_4(t) = \overline{F}(t)$

$M_6(t) = \overline{G}(t)\overline{F_1}(t)$ $M_9 = \overline{F_1}(t)\overline{H}(t)$ $M_{14}(t) = \overline{F}(t)\overline{H}(t)$

$M_7(t) = \overline{F_1}(t)$

$M_{10}(t) = \overline{F_1}(t)\overline{G_1}(t)$

$M_{17}(t) = \overline{F}(t)\overline{G_1}(t)$

Taking relation (4) for L.T. as well as explaining for $A_0^*(s)$.The system's steady-state availability is provided as:

$$A_0(\infty) = \lim_{s\to\infty} sA_0^*(s) = \frac{N_1}{D_1} \qquad \ldots(5)$$

where

$N_1 = \{[\mu_0 + p_{0,1}(t)\mu_1 + p_{0,3}\mu_3][1-L_1] + \{p_{0,3}(t)p_{3,4}(t)[\mu_4 + p_{4,6}(t)(\mu_6 + p_{6,7}(t)\mu_7 + p_{6,7}(t)p_{7,9}(t)\mu_9)]\} + p_{0,3}(t)p_{3,4}(t)p_{4,6}(t)p_{6,7}(t)p_{7,9}(t)p_{9,10}(t)\mu_{10}\} \div [1-L_1]$

$D_1 = \{[\mu_0 + p_{0,1}(t)\mu_1 + p_{0,3}(t)\mu_3][1-L_1] + \{p_{0,3}(t)p_{3,4}(t)[\mu_4 + p_{4,6}(t)(\mu_6 + p_{6,7}(t)\mu_7)]\} + p_{0,3}(t) p_{3,4}(t)p_{4,6}(t)p_{6,7}(t)p_{7,9}(t)[\mu_9 + p_{9,10}(t)\mu_{10}] + p_{0,3}(t)p_{3,4}(t)p_{4,6}(t)p_{6,7}(t)p_{7,9}(t)p_{9,12}(t)p_{12,14}(t)[\mu_{14} + p_{14,17}\mu_{17}]\} \div [1-L_1]$

where

$L_1 = [p_{4,6}(t)p_{6,7}(t)p_{7,9}(t)p_{9,4}(t) + p_{7,9}(t)p_{9,10}(t)p_{10,7}(t)]$

7. Busy Period Analysis of Servers

(i) Busy period of ordinary server for repair

Suppose $B_{i1}^{re}(t)$ represents probabilities that at an instant *t*, the server is busy in repair specified that at *t=0*, regenerative state *i* is entered by the system. For $B_{i1}^{re}(t)$, the recursive relations are as follows:

$$B_{i1}^{re}(t) = W_{(i)} + \sum q_{i,j}^{n}(t) © B_{j}^{re}(t) \qquad \ldots(6)$$

Where, *j* represents a subsequent regenerative state where through transitions $n \geq 1$ (natural numbers) transfer is made by state *i*.

$W_3(t) = [F(t) + (f(t) © 1)]\overline{G}(t)$

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$\overline{W_6}(t) = F_1(t) + [f_1(t) © 1]\overline{G}(t)$

The server's busy period for doing repair is given by:

$$B_1^{re} = \lim_{s\to 0} sB_1^{re*}(s) = \frac{N_2}{D_2}$$

where $D_2 = D_1$

$$B_1^{RE} = \{[p_{0,3}(t)w_3(1-L_1) + p_{0,3}(t)p_{3,4}(t)w_6]/[1-L_1]\} \div D_1$$

(ii) Busy period ordinary server for preventive maintenance

Suppose $B_{i1}^{pm}(t)$ represents probabilities that at an instant *t*, the server is busy in PM specified that at *t=0,* regenerative state *i* is entered by the system. For $B_{i1}^{PM}(t)$, the recursive relations are as follows:

$$B_{i1}^{pm}(t) = W_{(i)} + \sum q_{i,j}^{n}(t) © B_j^{pm}(t) \qquad \ldots(7)$$

Where, j represents a subsequent regenerative state where through transitions $n \geq 1$ (natural numbers) transfer is made by state i.

$W_1(t) = G_2(t)\overline{F(t)}$

The server's busy period for doing repair is given by:

$$B_1^{PM} = \lim_{s\to 0} sB_1^{PM*}(s) = \frac{N_3}{D_2}$$

where $D_2 = D_1$

$$B_1^{PM} = [p_{0,1}(t)w_1] \div D_1$$

(iii) Busy period of expert server for repair.

Suppose $B_{i2}^{re}(t)$ represents probabilities that at an instant *t,* the server is busy in repair specified that at *t=0*, regenerative state *i* is entered by the system. For $B_{i2}^{re}(t)$, the recursive relations are as follows:

$$B_{i2}^{re}(t) = W_{(i)} + \sum q_{i,j}^{n}(t) © B_j^{re}(t) \qquad \ldots(8)$$

Where, *j* represents a subsequent regenerative state where through $n \geq 1$ (natural numbers) transitions transfer is made by state i.

$\overline{W_{10}}(t) = F_1(t) + [f_1(t) © 1]\overline{G_1}(t)$ $\qquad W_{17} = [\overline{F(t)} + (f(t) © 1)]\overline{G_1(t)}$

The server's busy period for doing repair is given by:

$$B_2^{re} = \lim_{s\to 0} sB_2^{re*}(s) = \frac{N_4}{D_2}$$

where $D_2 = D_1$ and

$$B_2^{re} = \{[p_{0,3}(t)p_{3,4}(t)(p_{6,7}(t)p_{9,10}(t)w_{10} + p_{6,7}(t)p_{9,12}(t)p_{12,14}(t)p_{14,17}(t)w_{17})]/[1-L_1]\} \div D_1$$

(iv) Busy period of expert server for inspection.

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Suppose $B^{In}_{i2}(t)$ represents probabilities that at an instant t, the server is busy in repair specified that at *t=0,* regenerative state i is entered by the system. For $B^{In}_{i2}(t)$, the recursive relations are as follows:

$$B^{In}_{i2}(t) = W_{(i)} + \sum q^{n}_{i,j}(t) © B^{in}_{j}(t) \qquad \ldots(9)$$

Where, j represents a subsequent regenerative state where through transitions $n \geq 1$ (natural numbers) transfer is made by state i.

$$W_9(t) = \overline{F_1(t)}H(t) + [f_1(t) © 1]\overline{H(t)} + [f_1(t) © ph(t) © 1]\overline{G_1(t)}$$

$$W_{14}(t) = \overline{F(t)}H(t) + [f(t) © 1]\overline{H(t)} + [f(t) © ph(t) © 1]\overline{G_1(t)}$$

The server's busy period for doing Inspection is given by:

$$B^{In}_2 = \lim_{s \to 0} sB^{In*}_2(s) = \frac{N_5}{D_2}$$

where $D_2 = D_1$ Thus

$$B^{In}_2 = \{(p_{0,3}(t)p_{3,4}(t)p_{6,7}(t)[w_9 + p_{9,12}(t)p_{12,14}(t)p_{14,17}(t)w_{17}])/[1-L_1]\} \div D_1$$

8. Expected Number of Visits by the Server's

(i) No. of visits of ordinary server for repair.

Suppose $N^{re}_{i1}(t)$ represents the server's expected visits number in *(0,t)* provided that at *t=0,* regenerative state i is entered by the system. For $N^{re}_{i1}(t)$ recursive relation is as follows:

$$N^{re}_{i1}(t) = \sum_j Q_{i,j}(t) ® [\delta_j + N^{re}_j(t)] \qquad \ldots(10)$$

Where, $\delta_j = 1$, if j represents the regenerative state where in a fresh job is done by the server, or else $\delta_j = 0$. By using the above (10) recursive relation for the total number of visit/unit time is as:

$$N^{re}_1 = \lim_{s \to 0} N_1(s) = \frac{N_6}{D_3} \qquad \ldots(11)$$

where

$$N_6 = [p_{0,3}(t)(1-L_1) + p_{0,3}(t)p_{3,4}(t)p_{4,6}(t)]/[1-L_1]$$

and

$D_3 = D_1$ And D_1 is mentioned already.

(ii) No. of visits of ordinary server for PM.

Suppose $N^{pm}_{i1}(t)$ represents the server's expected visits number in *(0, t)* provided that at *t=0*, regenerative state i is entered by the system. For $N^{pm}_{i1}(t)$ recursive relation is as follows:

$$N^{pm}_{i1}(t) = \sum_j Q_{i,j}(t) ® [\delta_j + N^{pm}_j(t)] \qquad \ldots(12)$$

Where, $\delta_j = 1$, if j represents the regenerative state where in a fresh job is done by the server,

or else $\delta_j = 0$. By using the above (12) recursive relation for the total number of visit/unit time is as:

$$\lim_{s\to 0} \tilde{N}_1^{pm}(s) = \frac{N_7}{D_3} \quad \ldots(13)$$

where

$N_7 = p_{0,1}(t)$ and

$D_3 = D_1$ And D_1 is mentioned already.

(iii) No. of visits of expert server for repair.

Suppose $N_{i2}^{re}(t)$ represents the server's expected visits number in *(0,t)* provided that at *t=0*, regenerative state *i* is entered by the system. For $N_{i2}^{re}(t)$ recursive relation is as follows:

$$N_{i2}^{re}(t) = \sum_j Q_{i,j}(t) \circledR [\delta_j + N_j^{re}(t)] \quad \ldots(14)$$

Where, $\delta_j = 1$, if j represents the regenerative state where in a fresh job is done by the server,

or else $\delta_j = 0$. By using the above (14) recursive relation for the total number of visit/unit time is as:

$$\lim_{s\to 0} \tilde{N}_2^{re}(s) = \frac{N_8}{D_3} \quad \ldots(15)$$

where

$N_8 = p_{0,3}(t)\, p_{3,4}(t)\, p_{6,7}(t)\, p_{7,9}(t)[\, p_{9,10}(t) + p_{9,12}(t)\, p_{12,14}(t)\, p_{14,17}(t)]$

and

$D_3 = D_1$ And D_1 is mentioned already.

(iv) No. of visits of expert server for inspection.

Suppose $N_{i2}^{In}(t)$ represents the server's expected visits number in *(0,t)* provided that at *t=0*, regenerative state *i* is entered by the system. For $N_{i2}^{In}(t)$ recursive relation is as follows:

$$N_{i2}^{In}(t) = \sum_j Q_{i,j}(t) \circledR [\delta_j + N_j^{in}(t)] \quad \ldots(16)$$

Where, $\delta_j = 1$, if j represents the regenerative state where in a fresh job is done by the server,

or else $\delta_j = 0$. By using the above (16) recursive relation for the total number of visit/unit time is as:

$$\lim_{s\to 0} \tilde{N}_2^{in}(s) = \frac{N_9}{D_3} \quad \ldots(17)$$

where

$N_9 = p_{0,3}(t)\, p_{3,4}(t)\, p_{4,6}(t)\, p_{6,7}(t)\, p_{7,9}(t)$

and

$D_3 = D_1$ And D_1 is mentioned already.

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9. Profit Analysis

In a steady state, the profit incurred by the system model is provided by:

$$P_1 = C_1 A_0 - C_2 [B_1^{Re} + B_1^{PM}] - C_3 [B_2^{In} + B_2^{Re}] - C_4 [N_1^{Re} + N_1^{PM}] - C_5 [N_2^{Re} + N_2^{In}] \quad \ldots(18)$$

where

C_1: "The system's revenue per unit up-time"

C_2: "Cost per unit time for which ordinary server is busy in repair and preventive maintenance".

C_3: "Cost per unit time for which expert server is busy in repair and inspection".

C_4: "Cost per unit visit by the ordinary server for repair and preventive maintenance".

C_5: "Cost per unit visit by the expert server for repair and inspection".

10. Analytical Discussions:

"We assume that the distribution of failure times of units, repair times of units, inspection times, maximum operation time, and PM time is a Weibull distribution with two parameters to emphasize the significance of the results and to characterise the behavior of the MTSF, availability, and profit of the system".
Probability density function of Weibull distribution with two parameters is given by:

$$f(t) = k\lambda(\lambda t)^{k-1} \exp(-\lambda t)^k, \lambda > 0 \text{ and } t \geq 0$$

When $k = 1$, exponential distribution is followed as well as in case $k = 2$, Rayleigh distribution is followed. Suppose

$$f_1(t) = k\lambda_1(\lambda_1 t)^{k-1} \exp(-\lambda_1 t)^k, g(t) = kr(rt)^{k-1} \exp(rt)^k, g_1(t) = kr_1(r_1 t)^{k-1} \exp(r_1 t)^k$$

$$g_2(t) = kr_2(r_2 t)^{k-1} \exp(r_2 t)^k, h(t) = k\theta(\theta t)^{k-1} \exp(\theta t)^k, w(t) = kw(wt)^{k-1} \exp(wt)^k$$

Table: 1

This Table shows the Effect of failure rate and other parameters on MTSF for exponential distribution.

Failure Rate (λ)	**MTSF** (λ1=.13 r =.21, r1=.27, r2 =2.5, **p = 0.6, q = 0.4,** w = 10, θ = 20)	**MTSF (λ1=.17,** r=.21, r1=.27, r2 =2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**MTSF (r =.33,** λ1=.13, r1=.27, r2 = 2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**MTSF (r1=.36**, λ1=.13 r =.21, r2 = 2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**MTSF (λ1=.13**, r =.21, r1=.27, r2= 2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**MTSF** (λ1=.13, r =.21, r1=.27, **r2= 2.7**, p = 0.4, q = 0.6, w = 10, θ = 20)
0.01	155.7544967	149.7655399	171.3860187	163.6768068	159.5809581	160.7544967
0.02	79.31285905	76.27566081	88.36297133	84.06236756	81.89660654	82.31285905
0.03	53.74117382	51.71128253	60.60431547	57.41477816	55.89812058	56.04117382

0.04	40.8968557	39.38436054	46.66685879	44.01872679	42.83041129	42.89608557
0.05	33.14996841	31.95653752	38.2615333	35.92994183	34.94131507	35.14996841
0.06	27.95626711	26.98118621	32.62500431	30.49951166	29.64599934	29.95626711
0.07	22.53745245	23.40939427	28.57275147	26.59161034	25.83613094	26.03745245
0.08	19.94869538	20.71644642	25.5123348	23.63790625	22.95714658	23.14869538
0.09	17.92302336	18.61064227	23.1144802	21.32234950	20.70066173	20.92302336

Table: 2

This Table shows the Effect of failure rate and other parameters on Availability for exponential distribution.

Failure Rate (λ)	**Availability** (λ_1=.13 r =.21, r_1=.27, r_2 = 2.5, **p = 0.6, q = 0.4,** w = 10, θ = 20)	**Availability** (**λ_1=.17**, r =.21, r_1=.27, r_2 = 2.5, p = 0.6, q = 0.4, w = 10, θ = 20)	**Availability** (**r =.33**, λ_1=.13, r_1=.27, r_2 = 2.5, p = 0.6, q = 0.4, w = 10, θ = 20)	**Availability** (**r1=.36**, λ_1=.13 r =.21, r_2 = 2.5, p = 0.6, q = 0.4, w = 10, θ = 20)	**Availability** (**λ_1=.13** r =.21, r_1=.27, r_2=2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**Availability** (λ_1=.17, r =.21, r_1=.27, **r_2= 2.7**, p = 0.6, q = 0.4, w = 10, θ = 20)
0.01	0.999988113	0.999987554	0.999992562	0.999990582	0.999988206	0.999989123
0.02	0.999978368	0.999977178	0.99998556	0.999982691	0.999978505	0.999979012
0.03	0.999970336	0.999968489	0.999982601	0.999976043	0.999970483	0.999975401
0.04	0.999963688	0.999961186	0.99997852	0.999970417	0.999963822	0.999968345
0.05	0.999958169	0.99995503	0.99997224	0.99996564	0.999958273	0.999959631
0.06	0.999953575	0.99994983	0.999968524	0.99996157	0.999953639	0.999955484
0.07	0.999949748	0.999945431	0.999965901	0.999958096	0.999949764	0.999951063
0.08	0.999946558	0.999941706	0.999961431	0.999955126	0.999946522	0.999948304
0.09	0.999943902	0.999938551	0.999955563	0.999952583	0.99994381	0.999946534

Table: 3

This Table shows the Effect of failure rate and other parameters on Profit for exponential distribution.

Failure Rate (λ)	**Profit** (λ_1=.13 r =.21, r_1=.27, r_2 = 2.5, **p = 0.6, q = 0.4,** w = 10, θ = 20)	**Profit** (**λ_1=.17**, r =.21, r_1=.27, r_2 = 2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**Profit** (**r =.33**, λ_1=.13, r_1=.27, r_2 = 2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**Profit** (**r_1=.36**, λ_1=.13 r =.21, r_2 = 2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**Profit** (**λ_1=.13** r =.21, r_1=.27, r_2 = 2.5, p = 0.4, q = 0.6, w = 10, θ = 20)	**Profit** (λ_1=.17, r =.21, r_1=.27, **r_2 = 2.7**, p = 0.4, q = 0.6, w = 10, θ = 20)
0.01	3792.902516	3785.126338	3803.34785	3800.179538	3794.505796	3797.328535
0.02	3790.914976	3782.617114	3803.0832	3797.779914	3792.385203	3795.104529
0.03	3789.165043	3780.379554	3802.856312	3795.665257	3790.519655	3793.147759
0.04	3787.615582	3778.374272	3802.662206	3793.790565	3788.868793	3791.41573
0.05	3786.23679	3776.56912	3802.496746	3792.119829	3787.400331	3789.874486
0.06	3785.004473	3774.937599	3802.356467	3790.623887	3786.08813	3788.49657
0.07	3783.898783	3773.457676	3802.238441	3789.278873	3784.910801	3787.259536
0.08	3782.903287	3772.110876	3802.140176	3788.065064	3783.850674	3786.144856
0.09	3777.004262	3770.881596	3802.059538	3786.966017	3782.893021	3785.137102

10. Conclusion

The results for MTSF, availability and profit of two-unit cold Standby system have been obtained and tabulated presentations of these measures for arbitrary values to various parameter and costs have been made to highlight the behavior of MTSF, availability and profit in tables 1,2, and 3 respectively. This study reveals that MTSF, availability and profit of a system keep on decreasing with the increase of failure rates of the new unit and degraded unit while they increase with the increase in repair rates of the both unit (new unit and degraded unit) and preventive maintenance rate. Thus, the performance of a two-unit cold standby system can be improved by increasing the repair rate of the both unit (new unit and degraded unit) and preventive maintenance rate. From these tables it is also found that the availability and profit of the system increase with the replacement of degraded unit at its failure by new unit. So, it is concluded that a two-unit redundant system can be made more available and profitable to use by providing preventive maintenance to new unit and by providing priority in operation to new unit over degraded unit.

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References

1. Aggarwal, C., Ahlawat, N. and Malik, S.C. (2021a): Profit analysis of a standby repairable system with priority to preventive maintenance and rest of server between repairs. Journal of Reliability and Statistical Studies, 14(1), 57-81.
2. Asgarpoor, S. and Chan, G.K.(2001) Preventive maintenance with Markov processes. Proceedings of the 2001 North American Power Symposium. Vol. 510515.
3. Dawabsha, M. and Ruiz-Castro, J.E.(2020) A multi-state warm standby system with preventive maintenance, loss of units and an indeterminate multiple number of repairpersons. Computers & Industrial Engineering 142, 106348.
4. El-Sherbeny, M.S. and El-Said, K.M. (2010) Stochastic analysis of a two-unit cold standby system with two-stage repair and waiting time. Sankhya B 72, 1-10.
5. Goel, M. and Kumar, J.(2016) Availability and profit analysis of a two-unit cold standby system for general distribution. Cogent Mathematics 3(1), 1262937.
6. Kadyan, M.S. and kumar, J.(2012) Profit analysis of a system of non-identical units with degradation and replacement. International Journal of Computer Applications 40(3), 19-25.
7. Pham, H. and Wang, Y.(2011) A multi-objective optimization of imperfect preventive maintenance policy for dependent competing risk systems with hidden failure. IEEE Transactions on Reliability 60(4), 770-781.
8. Roussignol, M., Berenguer, C., Dieulle, L. and Grall, A. (2002) Continuous-time predictive-maintenance scheduling for a deteriorating system. IEEE transactions on reliability 51(2), 141-150.
9. Satsangi, S. K. and Agnihotri, R. K.(1996): Two-non-identical unit-system with priority-based repair and inspection. Microelectronics Reliability, 36(2) , 279-282.
10. Song, J., Ditlevsen, O.D. and Kiureghian, A.D.(2007) Availability, reliability and downtime of systems with repairable components. Reliability Engineering & System Safety 92(2), 231-242.
11. Singh, J., Chand, P. and Malik, S.C. (2008) Reliability and profit evaluation of an operating system with different repair strategy subject to degradation. Journal of Mathematics and System Sciences 4(1), 127-140.

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