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Fixed Point Theory for Fuzzy Metric Spaces with Integral and Control Function Applications

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Abstract
The central focus of this paper is to construct distinct fixed point theorems bounded by applying rational constraints within the domain of fuzzy metric topologies. Through these derivations, our research effectively extends and incorporates various recognized mathematical propositions established within contemporary peer-reviewed studies. Additionally, we offer pertinent applications to validate the theoretical assertions introduced here.
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1. Introduction

The initial development of fuzzy mathematical systems was spearheaded by Lofti A. Zadeh [13], whose 1965 introduction of fuzzy sets provided a robust mathematical avenue to model physical ambiguities. This pioneering work opened the door to a new paradigm in mathematical analysis, one that could accommodate uncertainty and vagueness inherent in real-world phenomena. Subsequently, diverse scholars capitalized on the topological properties of these sets to rigorously define non-crisp spaces, thereby laying the groundwork for a systematic exploration of fuzzy topology. Kramosil and Michalek [6] introduced the core architecture of fuzzy metric topologies in 1975, marking a crucial step toward formalizing the notion of distance in environments characterized by imprecision. Their contribution established a foundation upon which subsequent generations of mathematicians could build.

Building upon this mathematical scaffolding, Mariusz Grabiec [4] later adapted the classic fixed point theorems originally proved by Banach and Edelstein to fit this new fuzzy metric framework in 1988. This adaptation was pivotal, as fixed point theory serves as a cornerstone in nonlinear analysis and has extensive applications in differential equations, optimization, and dynamic systems. By extending these results into fuzzy contexts, Grabiec demonstrated the versatility and power of fuzzy metric spaces in addressing problems where classical approaches fall short.

The structural definition of fuzzy metric spaces saw a significant refinement in 1994 when George et al. [3] integrated continuous t-norms into the concept. This advancement not only enriched the theoretical landscape but also sparked the formulation of numerous fixed point theorems involving a wide array of generalized mapping classes, including compatible, weakly compatible, R-weakly compatible mappings, and implicit relations (as documented in [11, 12, 8, 9, 7, 1, 5, 2]). These developments underscored the adaptability of fuzzy metric spaces to accommodate increasingly complex relational structures, thereby broadening their applicability across mathematical and applied domains.

Concurrently, research spearheaded by R. K. Saini and Vishal Gupta [8, 9] concentrated on expansive-type functions and shared coincidence attributes involving R-weakly commuting mapping relations inside these fuzzy environments. Their work emphasized the nuanced interplay between mappings and metric structures, highlighting how fuzzy frameworks can capture subtleties overlooked in classical settings. Collectively, these contributions have transformed fuzzy metric spaces into a vibrant area of study, bridging abstract theory with practical applications.

The primary goal of the present manuscript is to broaden the seminal theories proposed by Mariusz Grabiec [4], simultaneously synthesizing and generalizing analogous outcomes frequently cited in current academic journals. By situating our work within this rich historical trajectory, we aim to contribute to the ongoing evolution of fuzzy fixed point theory, offering new insights that extend its scope and deepen its mathematical foundations.

2. Preliminaries

The following section details fundamental definitions and preparatory lemmas essential for the upcoming analytical arguments.

Definition 1 ([13]). Suppose Ω denotes an arbitrary universe. Within Ω, a fuzzy set A is characterized as a mapping that assigns every element in Ω to a specific membership value bounded strictly between the values of $[0,1]$ inclusive.

Definition 2 ([10]). A binary operation denoted by $*: [0,1] \times [0,1] \to [0,1]$ serves as a continuous t-norm exactly if the pairing $([0,1],*)$ operates as an abelian topological monoid having 1 as its multiplicative identity, satisfying the monotonicity rule $a * b \le c * d$ for any combination of values $a, b, c, d \in [0,1]$ where $a \le c$ and $b \le d$ hold true.

Definition 3 ([6]). We define the ordered combination $(\Omega, \mathcal{M}, *)$ as a fuzzy metric space assuming Ω constitutes an arbitrary collection of elements, $*$ acts as a continuous t-norm, and $\mathcal{M}$ embodies a fuzzy subset mapped across $\Omega^2 \times (0,\infty)$ that respects the subsequent axioms for any points $\vartheta, \beta, \gamma \in \Omega$ alongside parameters $t, s \in (0,\infty)$.

1. $\mathcal{M}(\vartheta, \beta, t) > 0$.
2. $\mathcal{M}(\vartheta, \beta, t) = 1$ iff $\vartheta = \beta$.
3. $\mathcal{M}(\vartheta, \beta, t) = \mathcal{M}(\beta, \vartheta, t)$.
4. $\mathcal{M}(\vartheta, \beta, t) * \mathcal{M}(\beta, \gamma, s) \le \mathcal{M}(\vartheta, \gamma, t+s)$.
5. The mapping $\mathcal{M}(\vartheta, \beta, \cdot): [0,\infty) \to [0,1]$ retains continuity.

Consequently, $\mathcal{M}$ acts as the fuzzy metric across the space Ω, with the expression $\mathcal{M}(\vartheta, \beta, t)$ quantifying the relative proximity connecting ϑ to β under parameter t.

Definition 4 ([4]). Assuming $(\Omega, \mathcal{M}, *)$ operates as a fuzzy metric topology, an unconstrained sequence $\{\vartheta_n\}$ located within Ω becomes considered to converge toward a specific point $\vartheta \in \Omega$ provided that the limit requirement $\lim_{n\to\infty} \mathcal{M}(\vartheta_n, \vartheta, t) = 1$ holds true for all continuous parameters $t > 0$.

Definition 5 ([4]). Within any specified fuzzy metric framework $(\Omega, \mathcal{M}, *)$, an ordered progression represented as $\{\vartheta_n\}$ inside Ω earns the classification of a Cauchy sequence whenever the equality $\lim_{n\to\infty} \mathcal{M}(\vartheta_{n+p}, \vartheta_n, t) = 1$ remains valid for strictly positive t and p.

Definition 6 ([4]). Any fuzzy metric space $(\Omega, \mathcal{M}, *)$ is formally recognized as complete assuming all internal Cauchy sequences inherently converge to a distinct point situated entirely inside that same space. Furthermore, it is deemed compact whenever one can deduce a convergent subsequence from any arbitrary sequence.

Lemma 7 ([4]). *The evaluation function $\mathcal{M}(\vartheta, \beta, \cdot)$ exhibits non-decreasing characteristics for any selected pair $\vartheta, \beta \in \Omega$.*

Lemma 8 ([7]). *Whenever an operational constant $k \in (0,1)$ can be identified fulfilling the inequality $\mathcal{M}(\vartheta, \beta, kt) \ge \mathcal{M}(\vartheta, \beta, t)$ across all spatial elements $\vartheta, \beta \in \Omega$ and temporal parameters $t \in (0,\infty)$, it logically dictates that $\vartheta = \beta$.*

We now present the proof of our primary findings.

3. Main Results

Theorem 9. *Let the triplet $(\Omega, \mathcal{M}, *)$ act as a strictly complete fuzzy metric structure accompanied by the self-mapping $\mathcal{F}: \Omega \to \Omega$ satisfying the conditions $\mathcal{M}(\vartheta, \beta, t) = 1$ and $\mathcal{M}(\mathcal{F}\vartheta, \mathcal{F}\beta, kt) \ge \lambda(\vartheta, \beta, t)$ where $\lambda(\vartheta, \beta, t) = \min\left\{\frac{\mathcal{M}(\beta, \mathcal{F}\beta, t)[1+\mathcal{M}(\vartheta, \mathcal{F}\vartheta, t)]}{[1+\mathcal{M}(\vartheta, \beta, t)]}, \mathcal{M}(\vartheta, \beta, t)\right\}$ for all arbitrary elements $\vartheta, \beta \in \Omega$ alongside the scaling factor $k \in (0,1)$. Given these exact parameters, the mapping $\mathcal{F}$ inevitably secures a strictly unique fixed point.*

Proof. Let us select some unconstrained initial element ϑ contained in Ω. From this point, we construct an iterative sequence $\{\vartheta_n\}$ inside Ω by establishing the relation $\mathcal{F}\vartheta_n = \vartheta_{n+1}$ across all natural numbers $n \in \mathbb{N}$.

Primary Claim. The generated succession $\{\vartheta_n\}$ qualifies as a Cauchy sequence.
By substituting the terms $\vartheta = \vartheta_{n-1}$ and $\beta = \vartheta_n$ directly into the bound (1.2), we infer

$$\mathcal{M}(\vartheta_n, \vartheta_{n+1}, kt) = \mathcal{M}(\mathcal{F}\vartheta_{n-1}, \mathcal{F}\vartheta_n, kt) \ge \lambda(\vartheta_{n-1}, \vartheta_n, t)$$

Now

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$$\lambda(\vartheta_{n-1},\vartheta_n,t)=\min\left\{\frac{\mathcal{M}(\vartheta_n,\mathcal{F}\vartheta_n,t)[1+\mathcal{M}(\vartheta_{n-1},\mathcal{F}\vartheta_{n-1},t)]}{[1+\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)]},\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)\right\},$$
$$\lambda(\vartheta_{n-1},\vartheta_n,t)=\min\left\{\frac{\mathcal{M}(\vartheta_n,\vartheta_{n+1},t)[1+\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)]}{[1+\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)]},\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)\right\},$$
$$\Rightarrow\lambda(\vartheta_{n-1},\vartheta_n,t)=\min\{\mathcal{M}(\vartheta_n,\vartheta_{n+1},t),\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)\}$$

If one assumes $\mathcal{M}(\vartheta_n,\vartheta_{n+1},t)\le\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)$, a direct look at equation (1.4) provides

$$\mathcal{M}(\vartheta_n,\vartheta_{n+1},kt)\ge\mathcal{M}(\vartheta_n,\vartheta_{n+1},t)$$

As an immediate consequence of Lemma (2), our primary proposition holds true without any extra steps. On the contrary, if the inequality $\mathcal{M}(\vartheta_n,\vartheta_{n+1},t)\ge\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)$ holds true, an examination of (1.4) demands

$$\mathcal{M}(\vartheta_n,\vartheta_{n+1},kt)\ge\mathcal{M}(\vartheta_{n-1},\vartheta_n,t)$$

Through the implementation of basic mathematical induction, we derive the following generalized formula valid across every sequence index n and any temporal parameter $t>0$:

$$\mathcal{M}(\vartheta_n,\vartheta_{n+1},kt)\ge\mathcal{M}\left(\vartheta,\vartheta_1,\frac{t}{k^{n-1}}\right)$$

Subsequently, choosing an arbitrary positive integer 's' gives

$$\mathcal{M}(\vartheta_n,\vartheta_{n+s},t)\ge\mathcal{M}\left(\vartheta_n,\vartheta_{n+1},\frac{t}{s}\right)*\cdots(s)\cdots*\mathcal{M}\left(\vartheta_{n+p-1},\vartheta_{n+p},\frac{t}{s}\right)$$

By using equation (1.5), we get

$$\mathcal{M}(\vartheta_n,\vartheta_{n+s},t)\ge\mathcal{M}\left(\vartheta,\vartheta_1,\frac{t}{sk^n}\right)*\cdots(s)\cdots*\mathcal{M}\left(\vartheta,\vartheta_1,\frac{t}{sk^n}\right)$$

Now taking $\lim_{n\to\infty}$ and using (1.1), we get

$$\lim_{n\to\infty}\mathcal{M}(\vartheta_n,\vartheta_{n+s},t)=1$$

Such an asymptotic evaluation definitively confirms that $\{\vartheta_n\}$ possesses the core characteristics of a Cauchy sequence. We shall denote its convergent limit as v.

Secondary Claim. The limit v acts as a fixed point for the mapping $\mathcal{F}$.

Observe the subsequent formulation:

$$\mathcal{M}(v,\mathcal{F}v,t)\ge\mathcal{M}(\mathcal{F}\vartheta_n,\mathcal{F}v,t)*\mathcal{M}(v,\vartheta_{n+1},t)$$
$$\ge\lambda\left(\vartheta_n,v,\frac{t}{2k}\right)*\mathcal{M}(v,\vartheta_{n+1},t)$$

Now

$$\lambda\left(\vartheta_n,v,\frac{t}{2k}\right)=\min\left\{\frac{\mathcal{M}\left(v,\mathcal{F}v,\frac{t}{2k}\right)\left[1+\mathcal{M}\left(\vartheta_n,\mathcal{F}\vartheta_n,\frac{t}{2k}\right)\right]}{\left[1+\mathcal{M}\left(v,\vartheta_n,\frac{t}{2k}\right)\right]},\mathcal{M}\left(v,\vartheta_n,\frac{t}{2k}\right)\right\}$$

Extracting the limit as $n\to\infty$ for the preceding relation while invoking property (1.1), we secure the following:

$$\lambda\left(v,v,\frac{t}{2k}\right)=\min\left\{\mathcal{M}\left(v,\mathcal{F}v,\frac{t}{2k}\right),1\right\}$$

Now if $\mathcal{M}(v,\mathcal{F}v,\frac{t}{2k})\ge 1$ then $\lambda(v,v,\frac{t}{2k})=1$. Therefore from (1.7) and using definition 3, we get v is a fixed point of $\mathcal{F}$. Now if $\mathcal{M}(v,\mathcal{F}v,\frac{t}{2k})\le 1$ then $\lambda(v,v,\frac{t}{2k})=\mathcal{M}(v,\mathcal{F}v,\frac{t}{2k})$. Hence from equation (1.7), we get

$$\mathcal{M}(v,\mathcal{F}v,t)\ge\mathcal{M}\left(v,\mathcal{F}v,\frac{t}{2k}\right)*\mathcal{M}(\vartheta_{n+1},v,t)$$

By executing $\lim_{n\to\infty}$ on inequality (1.8) and incorporating the stipulations of equation (1.1) alongside Lemma (2), it logically concludes that $\mathcal{F}v=v$.

Verification of Uniqueness: We must now confirm the singular nature of the fixed point v. Assuming the converse scenario, there would exist an alternate coordinate $w\in\Omega$ capable of satisfying $\mathcal{F}w=w$. Proceed to evaluate:

$$1\ge\mathcal{M}(w,v,t)=\mathcal{M}(\mathcal{F}w,\mathcal{F}v,t)\ge\lambda\left(w,v,\frac{t}{k}\right)$$

where

$$\lambda\left(w,v,\frac{t}{k}\right)=\min\left\{\frac{\mathcal{M}\left(v,\mathcal{F}v,\frac{t}{k}\right)\left[1+\mathcal{M}\left(w,\mathcal{F}w,\frac{t}{k}\right)\right]}{\left[1+\mathcal{M}\left(w,v,\frac{t}{k}\right)\right]},\mathcal{M}\left(w,v,\frac{t}{k}\right)\right\},$$
$$\lambda\left(w,v,\frac{t}{k}\right)=\min\left\{\frac{\mathcal{M}\left(v,v,\frac{t}{k}\right)\left[1+\mathcal{M}\left(w,w,\frac{t}{k}\right)\right]}{\left[1+\mathcal{M}\left(w,v,\frac{t}{k}\right)\right]},\mathcal{M}\left(w,v,\frac{t}{k}\right)\right\},$$
$$\lambda\left(w,v,\frac{t}{k}\right)=\min\left\{\frac{2}{\left[1+\mathcal{M}\left(w,v,\frac{t}{k}\right)\right]},\mathcal{M}\left(w,v,\frac{t}{k}\right)\right\}$$
$$=\min\{1,1\}$$
$$\Rightarrow\lambda\left(w,v,\frac{t}{k}\right)=1$$

Substituting this unity value into relation (1.9) rigidly forces $w=v$. This explicitly demonstrates that the transformation $\mathcal{F}$ harbors a strictly singular fixed point v, thus drawing the proof of Theorem 1 to a close. □

We introduce the family of mappings $\Phi=\{\phi\mid\phi:[0,1]\to[0,1]\}$, where each ϕ acts as a continuous mapping obeying $\phi(1)=1$, $\phi(0)=0$, alongside the strict inequality $\phi(a)>a$ universally applied across $0<a<1$.

Theorem 10. *Consider a structure $(\Omega,\mathcal{M},*)$ established as a completely bounded fuzzy metric space enclosing a mapping $\mathcal{F}:\Omega\to\Omega$ that upholds $\mathcal{M}(\vartheta,\beta,t)=1$ and $\mathcal{M}(\mathcal{F}\vartheta,\mathcal{F}\beta,kt)\ge\phi\{\lambda(\vartheta,\beta,t)\}$ where $\lambda(\vartheta,\beta,t)=\min\left\{\frac{\mathcal{M}(\beta,\mathcal{F}\beta,t)[1+\mathcal{M}(\vartheta,\mathcal{F}\vartheta,t)]}{[1+\mathcal{M}(\vartheta,\beta,t)]},\mathcal{M}(\vartheta,\beta,t)\right\}$ holding true for arbitrary $\vartheta,\beta\in\Omega$, operational constant $k\in(0,1)$, and any chosen function $\phi\in\Phi$. Within these bounds, $\mathcal{F}$ intrinsically possesses exactly one discrete fixed point.*

Proof. Proof. Given that ϕ is drawn from Φ, the strict constraint $\phi(a)>a$ inherently applies for any continuous value $0<a<1$. As a result, inequality (1.12) simplifies to

$$\mathcal{M}(\mathcal{F}\vartheta,\mathcal{F}\beta,kt)\ge\phi\{\lambda(\vartheta,\beta,t)\}\ge\lambda(\vartheta,\beta,t)$$

Relying heavily upon the sequential steps already outlined during the proof of Theorem 1, the intended mathematical resolution is immediately achieved. □

4. Applications

This segment illustrates practical applications that naturally stem from our core theoretical propositions. Let us analyze a continuous, strictly non-decreasing mapping $\Psi:[0,\infty)\to[0,\infty)$ expressly given by the integration $\Psi(t)=\int_0^t\phi(s)ds$ operating over any $t>0$ (which can also be written as $\Psi(t)=\int_0^t\phi(t)dt$

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where $t > 0$). Furthermore, this specific property insists that $\phi(\varepsilon) > 0$ for an arbitrary positive limit $\varepsilon > 0$, thereby confirming that the equality $\phi(t) = 0$ perfectly coincides with the absolute condition $t = 0$.

Theorem 11. *Allow* $(\Omega, \mathcal{M}, *)$ *to represent a rigorously complete fuzzy metric environment possessing an internal map* $\mathcal{F}: \Omega \to \Omega$ *dictated by the relations* $\mathcal{M}(\vartheta, \beta, t) = 1$

$$\int_0^{\mathcal{M}(\mathcal{F}\vartheta, \mathcal{F}\beta, kt)} \phi(t)dt \geq \int_0^{\lambda(\vartheta,\beta,t)} \phi(t)dt$$

where $\lambda(\vartheta, \beta, t) = \min\left\{\frac{\mathcal{M}(\beta,\mathcal{F}\beta,t)[1+\mathcal{M}(\vartheta,\mathcal{F}\vartheta,t)]}{[1+\mathcal{M}(\vartheta,\beta,t)]}, \mathcal{M}(\vartheta,\beta,t)\right\}$ *applicable to every paired combination* $\vartheta, \beta \in \Omega$, *continuous mapping* $\phi \in \Psi$, *and ratio* $k \in (0,1)$. *Under such premises, the self-map* $\mathcal{F}$ *definitively converges to a solitary fixed point.*

Proof. Proof. Setting the functional parameter to the constant $\phi(t) = 1$ and integrating the established logic of Theorem 1 effectively procures the required derivation. □

Theorem 12. *Suppose the system* $(\Omega, \mathcal{M}, *)$ *constitutes an entirely closed fuzzy metric space equipped with the self-mapping* $\mathcal{F}: \Omega \to \Omega$ *fulfilling* $\mathcal{M}(\vartheta, \beta, t) = 1$ $\int_0^{\mathcal{M}(\mathcal{F}\vartheta,\mathcal{F}\beta,kt)} \phi(t)dt \geq \phi\left\{\int_0^{\lambda(\vartheta,\beta,t)} \phi(t)dt\right\}$ *where* $\lambda(\vartheta, \beta, t) = \min\left\{\frac{\mathcal{M}(\beta,\mathcal{F}\beta,t)[1+\mathcal{M}(\vartheta,\mathcal{F}\vartheta,t)]}{[1+\mathcal{M}(\vartheta,\beta,t)]}, \mathcal{M}(\vartheta,\beta,t)\right\}$ *irrespective of the choices for* $\vartheta, \beta \in \Omega$, *scalar quantity* $k \in (0,1)$, *and respective functional bounds* $\phi \in \Psi$ *as well as* $\phi \in \Phi$. *Restricted by these conditions,* $\mathcal{F}$ *uniquely stabilizes at a single spatial point.*

Proof. Recognizing that the strict boundary condition $\phi(a) > a$ remains persistently active whenever $0 < a < 1$, the final theoretical deduction is trivially extracted by applying the framework of Theorem 3. □

Remark 13. The theoretical advancements presented in this article extend and generalize the foundational results established by Mariusz Grabiec [4], as well as numerous other contributions currently documented in the academic literature.

5. Conclusions

This research work successfully establishes novel fixed-point theorems for self-mappings within the framework of complete fuzzy metric spaces. By introducing a specialized rational contractive condition, we rigorously proved the existence and uniqueness of fixed points (Theorem 1) and further generalized these outcomes utilizing continuous regulatory control functions (Theorem 2). Furthermore, we demonstrated the mathematical versatility of our primary findings by extending them to integral-type mappings, yielding robust application-based results. The theoretical deductions derived herein not only refine the foundational principles initially proposed by Mariusz Grabiec but also substantially expand upon contemporary developments in fuzzy mathematical analysis. The methodologies and integral applications presented in this study provide a strong structural foundation for future explorations. Subsequent research could potentially extend these rational inequalities to investigate common coincidence points, multi-valued mappings, or adapt these integral control functions to generalized topological structures such as fuzzy b-metric spaces or intuitionistic fuzzy metric spaces.

References

1. Seong Hoon Cho et al. "On common fixed point theorems in fuzzy metric spaces". In: *International Mathematical Forum* 1.9–12 (2006), pp. 471–479.
2. Y. J. Cho, S. Sedghi, and N. Shobe. "Generalized fixed point theorems for Compatible mappings with some types in fuzzy metric spaces". In: *Chaos, Solitons and Fractals* 39 (2009), pp. 2233–2244.
3. George and P. Veeramani. "On some results in fuzzy metric spaces". In: *Fuzzy Sets and Systems* 64.3 (1994), pp. 395–399.
4. M. Grabiec. "Fixed Points in Fuzzy Metric Spaces". In: *Fuzzy Sets and Systems* 27.3 (1988), pp. 385–389.
5. V. Gregori and A. Sapena. "On Fixed-Point Theorems in Fuzzy Metric Spaces". In: *Fuzzy Sets and Systems* 125 (2002), pp. 245–252.
6. Kramosil and J. Michálek. "Fuzzy Metrics and Statistical Metric Spaces". In: *Kybernetika* 11.5 (1975), pp. 336–344.
7. S. N. Mishra, S. N. Sharma, and S. L. Singh. "Common fixed point of maps on fuzzy metric spaces". In: *International Journal of Mathematics and Mathematical Sciences* 17 (1994), pp. 253–258.
8. R. K. Saini and V. Gupta. "Fuzzy Version of Some Fixed Points Theorems On Expansion Type Maps in Fuzzy Metric Space". In: *Thai Journal of Mathematics* 5.2 (2007), pp. 245–252.
9. R. K. Saini and V. Gupta. "Common coincidence Points of R-Weakly Commuting Fuzzy Maps". In: *Thai Journal of Mathematics* 6.1 (2008), pp. 109–115.
10. Schweizer and A. Sklar. *Probabilistic Metric Spaces*. North-Holland Series in Probability and Applied Mathematics. New York: North-Holland Publishing Co., 1983.
11. R. Vasuki. "A common fixed point theorem in a fuzzy metric space". In: *Fuzzy Sets and Systems* 97 (1998), pp. 395–397.
12. R. Vasuki. "Common Fixed Points for R-Weakly Commuting Maps in Fuzzy Metric Spaces". In: *Indian Journal of Pure and Applied Mathematics* 30 (1999), pp. 419–423.
13. L. A. Zadeh. "Fuzzy Sets". In: *Information and Control 8.3 (1965), pp. 338–353.*