

A NOVEL DISTANCE BASED SIMILARITY MEASURE FOR PYTHAGOREAN NEUTROSOPHIC CUBIC SETS USING MADM APPROACH FOR AIR CONDITIONING MECHANISM

Berna Joyce L¹ & Elvina Mary L²

¹Research Scholar, PG & Research Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India
 Email: bernajoyceslr@gmail.com

²Assistant Professor, PG & Research Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.
 Email: elvinalawrence07@gmail.com

Corresponding Author: bernajoyceslr@gmail.com

ABSTRACT

A Pythagorean Neutrosophic Cubic Set [PNCS] is a hybrid of Interval Valued Pythagorean Neutrosophic Set (IVPNS) and Pythagorean Neutrosophic Set (PNS). The Distance Similarity metric on the Pythagorean Neutrosophic Cubic Set is proposed in this study. A generalized distance measure between PNCS and two distance-based similarity measures of PNCS were proposed. A PNCS similarity measure-based algorithm is then introduced. Finally, a demonstration of the MADM problem utilizing the distance similarity measure for air conditioning mechanism has been provided.

KEYWORDS Pythagorean Neutrosophic Cubic Set, Distance Measure, Similarity Measure, Generalized Similarity measure, Multiple Attribute Decision Making

1. INTRODUCTION

Zadeh [14] implemented the principle of fuzzy sets in 1965, which laid a solid foundation for fuzzy mathematics. A modified arrangement of fuzzy sets with interval valued function composition would be suggested immediately afterwards, in 1975, by Zadeh [15]. The designation Neutrosophic Sets (NS) first came into existence by Florentin Smarandache in 1995 [7]. Jun et al [4] pointed out Cubic Sets (CS) methodology in 2012. Neutrosophic Cubic Sets (NCS) got exposed by Chang Su Kim, Young Bae Jun, and Florentin Smarandache [5] in 2017. The continued development of the Pythagorean fuzzy set (PFS) was first launched by Yagar [9] in 2013. According to F. Khana, M. S. Ali Khana, M. Shahzada, and S. Abdullah [6], the Pythagorean Cubic Fuzzy Set (PCFS) came into being in 2019. R. Jhansi & K. Mohana [3] established the Pythagorean Neutrosophic Sets (PNS). Stephy et al put forth the Interval-Valued Neutrosophic Pythagorean Sets (IVNPS) [8]. Berna Joyce [1] pointed out a novel idea for Pythagorean Neutrosophic Cubic Sets (PNCS). The MADM method for NCS was highlighted by Jun Ye [10] in 2018. Ye [10][11][12][13] additionally mentioned the correlation coefficient and cross-entropy measure of single-valued NS. Then, in 2020, R. Janshi and Mohana K [2] demonstrated the Similarity Measure of PNS. Lastly, Ye [11] brought forward a similarity measure between INS and their use in multicriteria decision making. This study suggests a similarity Measure for the PNCS that is based on distance Measure. Two Distance-based Similarity Measures of PNCS are proposed, together with a Generalized Distance Measure between PNCS. Then we played with an application based on the Similarity Measure.

2. PRELIMINARIES

Definition 2.1[3] Let X be a non-empty set of the universe. A Pythagorean Neutrosophic set (PNS) with T and F are dependent Neutrosophic components A on X is an object of the form $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$ where $T_A, I_A, F_A : X \rightarrow [0,1]$ and $0 \leq (T_A(x))^2 + (I_A(x))^2 + (F_A(x))^2 \leq 2$ where $T_A(x)$ denotes the degree of membership, $I_A(x)$ denotes the degree of indeterminacy and $F_A(x)$ denotes the degree of non-membership.

Definition 2.2[8] Let X be a non-empty set of the universe, a Pythagorean Neutrosophic Interval Valued Set (PNIVS) with T and F are dependent Neutrosophic components on A in X is of the form, $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$ where $T_A(x) = [T_A^-(x), T_A^+(x)]$, $I_A(x) = [I_A^-(x), I_A^+(x)]$ and $F_A(x) = [F_A^-(x), F_A^+(x)]$.

Consider the mapping $T_A, I_A, F_A : X \rightarrow [0,1]$ and $0 \leq \left[\frac{T_A^-(x) + T_A^+(x)}{2} \right]^2 + \left[\frac{I_A^-(x) + I_A^+(x)}{2} \right]^2 + \left[\frac{F_A^-(x) + F_A^+(x)}{2} \right]^2 \leq 2$ where $T_A(x)$ denotes the degree of membership, $I_A(x)$ denotes the degree of indeterminacy and $F_A(x)$ denotes the degree of non-membership.

Definition 2.3[1] Let X be a non-empty set of the universe. A Pythagorean Neutrosophic Cubic Set (PNCS) can be defined as follows $\mathbb{A} = \{ \langle x, A(x), \lambda(x) \rangle : x \in X \}$ where $A(x)$ represent the Pythagorean Neutrosophic Interval valued set in X $\lambda(x)$ represent the Pythagorean Neutrosophic Set. PNCS can be denoted as a pair $\mathbb{A} = (A, \lambda)$.

3. DISTANCE BASED SIMILARITY MEASURE BETWEEN PYTHAGOREAN NEUTROSOPHIC CUBIC SET

3.1. Distance Measure of Pythagorean Neutrosophic Cubic Set:

Definition 3.1.1 For any PNCS \mathbb{A} and \mathbb{B} in a universe of discourse X which are denoted by

$$\mathbb{A}_i = \langle A_i, \lambda_i \rangle = \left\{ \langle x_i, [T_A^-(x_i), T_A^+(x_i)], [I_A^-(x_i), I_A^+(x_i)], [F_A^-(x_i), F_A^+(x_i)] \rangle \text{ and } \langle \lambda_T(x_i), \lambda_I(x_i), \lambda_F(x_i) \rangle : x_i \in X \right\}$$

$$\mathbb{B}_i = \langle B_i, \gamma_i \rangle = \left\{ \langle x_i, [T_B^-(x_i), T_B^+(x_i)], [I_B^-(x_i), I_B^+(x_i)], [F_B^-(x_i), F_B^+(x_i)] \rangle \text{ and } \langle \gamma_T(x_i), \gamma_I(x_i), \gamma_F(x_i) \rangle : x_i \in X \right\}$$

. Let $w_i (i = 1, 2, \dots, n)$ be the weight of any element x_i with $w_i \geq 0$ and $\sum_{i=1}^n w_i = 1$. Then we define the Generalized PNC weighted distance measure (GPNCWDM):

$$d_p(\mathbb{A}, \mathbb{B}) = \frac{1}{9} \sum_{i=1}^n w_i \left\{ \left[\left| (T_A^-(x_i))^2 - (T_B^-(x_i))^2 \right|^p + \left| (I_A^-(x_i))^2 - (I_B^-(x_i))^2 \right|^p + \left| (F_A^-(x_i))^2 - (F_B^-(x_i))^2 \right|^p \right] + \left[\left| (T_A^+(x_i))^2 - (T_B^+(x_i))^2 \right|^p + \left| (I_A^+(x_i))^2 - (I_B^+(x_i))^2 \right|^p + \left| (F_A^+(x_i))^2 - (F_B^+(x_i))^2 \right|^p \right] + \left[\left| \lambda_T^2(x_i) - \gamma_T^2(x_i) \right|^p + \left| \lambda_I^2(x_i) - \gamma_I^2(x_i) \right|^p + \left| \lambda_F^2(x_i) - \gamma_F^2(x_i) \right|^p \right] \right\}^{\frac{1}{p}} \rightarrow (1)$$

where $p > 0$. When $p = 1, 2$ we can obtain the PNC weighted Hamming distance (PNCWHDM) and Pythagorean Neutrosophic Cubic weighted Euclidean distance (PNCWEDM) respectively as follows:

$$d_1(\mathbb{A}, \mathbb{B}) = \frac{1}{9} \sum_{i=1}^n w_i \left\{ \left[\left| (T_A^-(x_1))^2 - (T_B^-(x_1))^2 \right| + \left| (I_A^-(x_1))^2 - (I_B^-(x_1))^2 \right| + \left| (F_A^-(x_1))^2 - (F_B^-(x_1))^2 \right| \right] + \left[\left| (T_A^+(x_1))^2 - (T_B^+(x_1))^2 \right| + \left| (I_A^+(x_1))^2 - (I_B^+(x_1))^2 \right| + \left| (F_A^+(x_1))^2 - (F_B^+(x_1))^2 \right| \right] + \left[\left| \lambda_T^2(x_1) - \gamma_T^2(x_1) \right| + \left| \lambda_I^2(x_1) - \gamma_I^2(x_1) \right| + \left| \lambda_F^2(x_1) - \gamma_F^2(x_1) \right| \right] \right\} \rightarrow (2)$$

$$d_2(\mathbb{A}, \mathbb{B}) = \frac{1}{9} \sum_{i=1}^n w_i \left\{ \left| (T_A^-(x_2))^2 - (T_B^-(x_2))^2 \right|^2 + \left| (I_A^-(x_2))^2 - (I_B^-(x_2))^2 \right|^2 + \left| (F_A^-(x_2))^2 - (F_B^-(x_2))^2 \right|^2 + \right. \\ \left. \left| (T_A^+(x_2))^2 - (T_B^+(x_2))^2 \right|^2 + \left| (I_A^+(x_2))^2 - (I_B^+(x_2))^2 \right|^2 + \left| (F_A^+(x_2))^2 - (F_B^+(x_2))^2 \right|^2 + \right. \\ \left. |\lambda_T^2(x_2) - \gamma_T^2(x_2)|^2 + |\lambda_I^2(x_2) - \gamma_I^2(x_2)|^2 + |\lambda_F^2(x_2) - \gamma_F^2(x_2)|^2 \right\}^{\frac{1}{2}} \rightarrow (3)$$

∴ Equ (2) & (3) are the special cases of (1). Then for the distance measure, we have the following Proposition.

Proposition 3.1.2 The above- defined distance $d_p(\mathbb{A}, \mathbb{B})$ for $p > 0$ satisfies the following properties

1. $0 \leq d_p(\mathbb{A}, \mathbb{B}) \leq 1$
2. $d_p(\mathbb{A}, \mathbb{B}) = 0$ if and only if $\mathbb{A} = \mathbb{B}$
3. $d_p(\mathbb{A}, \mathbb{B}) = d_p(\mathbb{B}, \mathbb{A})$
4. If $\mathbb{A} \subseteq \mathbb{B} \subseteq \mathbb{O}$, where \mathbb{O} is a PNC-set in X , then

$$d_p(\mathbb{A}, \mathbb{O}) \geq d_p(\mathbb{A}, \mathbb{B}) \text{ and } d_p(\mathbb{A}, \mathbb{O}) \geq d_p(\mathbb{B}, \mathbb{O}).$$

Proof: It's apparent that $d_p(\mathbb{A}, \mathbb{B})$ fulfills the requirements (1)-(3). ∴ We can only prove (4).

Let $\mathbb{A} \subseteq \mathbb{B} \subseteq \mathbb{O}$, then $T_A^-(x_i) \leq T_B^-(x_i) \leq T_O^-(x_i)$; $T_A^+(x_i) \leq T_B^+(x_i) \leq T_O^+(x_i)$;

$I_A^-(x_i) \geq I_B^-(x_i) \geq I_O^-(x_i)$; $I_A^+(x_i) \geq I_B^+(x_i) \geq I_O^+(x_i)$; $F_A^-(x_i) \geq F_B^-(x_i) \geq F_O^-(x_i)$; $F_A^+(x_i) \geq F_B^+(x_i) \geq F_O^+(x_i)$; $\lambda_T(x_i) \leq \gamma_T(x_i) \leq \alpha_T(x_i)$; $\lambda_I(x_i) \leq \gamma_I(x_i) \leq \alpha_I(x_i)$; $\lambda_F(x_i) \leq \gamma_F(x_i) \leq \alpha_F(x_i)$ for $x_i \in X$. Also, the same condition can be applied while squaring each terms.

$$(T_A^-(x_i))^2 \leq (T_B^-(x_i))^2 \leq (T_O^-(x_i))^2; (T_A^+(x_i))^2 \leq (T_B^+(x_i))^2 \leq (T_O^+(x_i))^2;$$

$$(I_A^-(x_i))^2 \geq (I_B^-(x_i))^2 \geq (I_O^-(x_i))^2; (I_A^+(x_i))^2 \geq (I_B^+(x_i))^2 \geq (I_O^+(x_i))^2;$$

$$(F_A^-(x_i))^2 \geq (F_B^-(x_i))^2 \geq (F_O^-(x_i))^2; (F_A^+(x_i))^2 \geq (F_B^+(x_i))^2 \geq (F_O^+(x_i))^2;$$

$$|\lambda_T^2(x_i) - \gamma_T^2(x_i)| \leq |\alpha_T^2(x_i)|; |\lambda_I^2(x_i) - \gamma_I^2(x_i)| \leq |\alpha_I^2(x_i)|; |\lambda_F^2(x_i) - \gamma_F^2(x_i)| \leq |\alpha_F^2(x_i)|$$

Then we obtain the following relations:

$$\left| (T_A^-(x_i))^2 - (T_B^-(x_i))^2 \right|^p \leq \left| (T_A^-(x_i))^2 - (T_O^-(x_i))^2 \right|^p; \left| (T_B^-(x_i))^2 - (T_O^-(x_i))^2 \right|^p \leq \left| (T_A^-(x_i))^2 - (T_O^-(x_i))^2 \right|^p$$

$$\left| (T_A^+(x_i))^2 - (T_B^+(x_i))^2 \right|^p \leq \left| (T_A^+(x_i))^2 - (T_O^+(x_i))^2 \right|^p; \left| (T_B^+(x_i))^2 - (T_O^+(x_i))^2 \right|^p \leq \left| (T_A^+(x_i))^2 - (T_O^+(x_i))^2 \right|^p$$

$$\left| (I_A^-(x_i))^2 - (I_B^-(x_i))^2 \right|^p \leq \left| (I_A^-(x_i))^2 - (I_O^-(x_i))^2 \right|^p; \left| (I_B^-(x_i))^2 - (I_O^-(x_i))^2 \right|^p \leq \left| (I_A^-(x_i))^2 - (I_O^-(x_i))^2 \right|^p$$

$$\left| (I_A^+(x_i))^2 - (I_B^+(x_i))^2 \right|^p \leq \left| (I_A^+(x_i))^2 - (I_O^+(x_i))^2 \right|^p; \left| (I_B^+(x_i))^2 - (I_O^+(x_i))^2 \right|^p \leq \left| (I_A^+(x_i))^2 - (I_O^+(x_i))^2 \right|^p$$

$$\left| (F_A^-(x_i))^2 - (F_B^-(x_i))^2 \right|^p \leq \left| (F_A^-(x_i))^2 - (F_O^-(x_i))^2 \right|^p; \left| (F_B^-(x_i))^2 - (F_O^-(x_i))^2 \right|^p \leq \left| (F_A^-(x_i))^2 - (F_O^-(x_i))^2 \right|^p$$

$$\left| (F_A^+(x_i))^2 - (F_B^+(x_i))^2 \right|^p \leq \left| (F_A^+(x_i))^2 - (F_O^+(x_i))^2 \right|^p; \left| (F_B^+(x_i))^2 - (F_O^+(x_i))^2 \right|^p \leq \left| (F_A^+(x_i))^2 - (F_O^+(x_i))^2 \right|^p$$

$$|\lambda_T^2(x_i) - \gamma_T^2(x_i)|^p \leq |\lambda_T^2(x_i) - \alpha_T^2(x_i)|^p; |\gamma_T^2(x_i) - \alpha_T^2(x_i)|^p \leq |\lambda_T^2(x_i) - \alpha_T^2(x_i)|^p$$

$$|\lambda_I^2(x_i) - \gamma_I^2(x_i)|^p \leq |\lambda_I^2(x_i) - \alpha_I^2(x_i)|^p; |\gamma_I^2(x_i) - \alpha_I^2(x_i)|^p \leq |\lambda_I^2(x_i) - \alpha_I^2(x_i)|^p$$

$$|\lambda_F^2(x_i) - \gamma_F^2(x_i)|^p \leq |\lambda_F^2(x_i) - \alpha_F^2(x_i)|^p; |\gamma_F^2(x_i) - \alpha_F^2(x_i)|^p \leq |\lambda_F^2(x_i) - \alpha_F^2(x_i)|^p$$

Hence,

$$\left| (T_A^-(x_i))^2 - (T_B^-(x_i))^2 \right|^p + \left| (T_A^+(x_i))^2 - (T_B^+(x_i))^2 \right|^p + \left| (I_A^-(x_i))^2 - (I_B^-(x_i))^2 \right|^p + \left| (I_A^+(x_i))^2 - (I_B^+(x_i))^2 \right|^p \\ \left| (F_A^-(x_i))^2 - (F_B^-(x_i))^2 \right|^p + \left| (F_A^+(x_i))^2 - (F_B^+(x_i))^2 \right|^p + |\lambda_T^2(x_i) - \gamma_T^2(x_i)|^p + |\lambda_I^2(x_i) - \gamma_I^2(x_i)|^p + |\lambda_F^2(x_i) - \gamma_F^2(x_i)|^p \\ \leq \left| (T_A^-(x_i))^2 - (T_O^-(x_i))^2 \right|^p + \left| (T_A^+(x_i))^2 - (T_O^+(x_i))^2 \right|^p + \left| (I_A^-(x_i))^2 - (I_O^-(x_i))^2 \right|^p + \left| (I_A^+(x_i))^2 - (I_O^+(x_i))^2 \right|^p + \\ \left| (F_A^-(x_i))^2 - (F_O^-(x_i))^2 \right|^p + \left| (F_A^+(x_i))^2 - (F_O^+(x_i))^2 \right|^p + |\lambda_T^2(x_i) - \alpha_T^2(x_i)|^p + |\lambda_I^2(x_i) - \alpha_I^2(x_i)|^p + \\ |\lambda_F^2(x_i) - \alpha_F^2(x_i)|^p$$

and

$$\left| (T_B^-(x_i))^2 - (T_O^-(x_i))^2 \right|^p + \left| (T_B^+(x_i))^2 - (T_O^+(x_i))^2 \right|^p + \left| (I_B^-(x_i))^2 - (I_O^-(x_i))^2 \right|^p + \left| (I_B^+(x_i))^2 - (I_O^+(x_i))^2 \right|^p + \\ \left| (F_B^-(x_i))^2 - (F_O^-(x_i))^2 \right|^p + \left| (F_B^+(x_i))^2 - (F_O^+(x_i))^2 \right|^p + |\gamma_T^2(x_i) - \alpha_T^2(x_i)|^p + \\ |\gamma_I^2(x_i) - \alpha_I^2(x_i)|^p + |\gamma_F^2(x_i) - \alpha_F^2(x_i)|^p \\ \leq \left| (T_A^-(x_i))^2 - (T_O^-(x_i))^2 \right|^p + \left| (T_A^+(x_i))^2 - (T_O^+(x_i))^2 \right|^p + \left| (I_A^-(x_i))^2 - (I_O^-(x_i))^2 \right|^p + \left| (I_A^+(x_i))^2 - (I_O^+(x_i))^2 \right|^p + \\ \left| (F_A^-(x_i))^2 - (F_O^-(x_i))^2 \right|^p + \left| (F_A^+(x_i))^2 - (F_O^+(x_i))^2 \right|^p + |\lambda_T^2(x_i) - \alpha_T^2(x_i)|^p + |\lambda_I^2(x_i) - \alpha_I^2(x_i)|^p + \\ |\lambda_F^2(x_i) - \alpha_F^2(x_i)|^p$$

Combining the above inequalities with the above defined distance formulas, we can obtain,

$$d_p(\mathbb{A}, \mathbb{O}) \geq d_p(\mathbb{A}, \mathbb{B}) \text{ and } d_p(\mathbb{A}, \mathbb{O}) \geq d_p(\mathbb{B}, \mathbb{O}) \text{ for } p > 0.$$

∴ Property (4) is satisfied. This completes the proof.

3.2 Similarity Measure of Pythagorean Neutrosophic Cubic Set:

The similarity and distance measures are complementary. When the first increases, the second decreases. Normalized distance measure and similarity measure are dual concepts.

Thus $S(\mathbb{A}, \mathbb{B}) = 1 - d(\mathbb{A}, \mathbb{B})$ are vice versa. The properties of distance measure below are complementary to those of similarity measure.

Proposition 3.2.1

Let \mathbb{A} and \mathbb{B} be two Pythagorean Neutrosophic Cubic Sets in a universe of discourse X . $S(\mathbb{A}, \mathbb{B})$ is called a Pythagorean Neutrosophic Cubic Similarity Measure(PNCSM), which should satisfy the following properties

1. $0 \leq S(A, \mathbb{B}) \leq 1$
2. $S(A, \mathbb{B}) = 0$ iff $A = \mathbb{B}$
3. $S(A, \mathbb{B}) = S(\mathbb{B}, A)$
4. If $A \subseteq \mathbb{B} \subseteq \mathbb{O}$, where \mathbb{O} is a PNC-set in X , then

$S(A, \mathbb{O}) \geq S(A, \mathbb{B})$ and $S(A, \mathbb{O}) \geq S(\mathbb{B}, \mathbb{O})$

Assume that there are two Pythagorean Neutrosophic Cubic-Sets A and \mathbb{B} in X where

$A = \{x, [T_A^-(x), T_A^+(x)], [I_A^-(x), I_A^+(x)], [F_A^-(x), F_A^+(x)], (\lambda_T(x), \lambda_I(x), \lambda_F(x)) : x \in X\}$ and

$\mathbb{B} = \{x, [T_B^-(x), T_B^+(x)], [I_B^-(x), I_B^+(x)], [F_B^-(x), F_B^+(x)], (\gamma_T(x), \gamma_I(x), \gamma_F(x)) : x \in X\}$

Thus according to the relationship between the distance and the similarity measure, we obtain the following Pythagorean Neutrosophic Cubic similarity measure.

$S_1(A, \mathbb{B}) = 1 - d_p(A, \mathbb{B})$

$$= 1 - \left\{ \frac{1}{9} \sum_{i=1}^n w_i \left[\left| (T_A^-(x_i))^2 - (T_B^-(x_i))^2 \right|^p + \left| (I_A^-(x_i))^2 - (I_B^-(x_i))^2 \right|^p + \left| (F_A^-(x_i))^2 - (F_B^-(x_i))^2 \right|^p + \left| \lambda_T^2(x_i) - \gamma_T^2(x_i) \right|^p + \left| \lambda_I^2(x_i) - \gamma_I^2(x_i) \right|^p + \left| \lambda_F^2(x_i) - \gamma_F^2(x_i) \right|^p \right]^{\frac{1}{p}} \right\} \rightarrow (4)$$

Obviously, we can prove that $S_1(A, \mathbb{B})$ satisfies the properties (1) – (4) in Proposition 3.3 by the relationship between the distance and the similarity measure and the proof of proposition 3.2 which is omitted here.

Futhermore, we can also propose another PNC similarity measure.

$$S_2(A, \mathbb{B}) = \frac{1 - d_p(A, \mathbb{B})}{1 + d_p(A, \mathbb{B})}$$

$$= \frac{1 - \left\{ \frac{1}{9} \sum_{i=1}^n w_i \left[\left| (T_A^-(x_i))^2 - (T_B^-(x_i))^2 \right|^p + \left| (I_A^-(x_i))^2 - (I_B^-(x_i))^2 \right|^p + \left| (F_A^-(x_i))^2 - (F_B^-(x_i))^2 \right|^p + \left| \lambda_T^2(x_i) - \gamma_T^2(x_i) \right|^p + \left| \lambda_I^2(x_i) - \gamma_I^2(x_i) \right|^p + \left| \lambda_F^2(x_i) - \gamma_F^2(x_i) \right|^p \right]^{\frac{1}{p}} \right\}}{1 + \left\{ \frac{1}{9} \sum_{i=1}^n w_i \left[\left| (T_A^-(x_i))^2 - (T_B^-(x_i))^2 \right|^p + \left| (I_A^-(x_i))^2 - (I_B^-(x_i))^2 \right|^p + \left| (F_A^-(x_i))^2 - (F_B^-(x_i))^2 \right|^p + \left| \lambda_T^2(x_i) - \gamma_T^2(x_i) \right|^p + \left| \lambda_I^2(x_i) - \gamma_I^2(x_i) \right|^p + \left| \lambda_F^2(x_i) - \gamma_F^2(x_i) \right|^p \right]^{\frac{1}{p}} \right\}} \rightarrow (5)$$

Then similarity measure $S_2(A, \mathbb{B})$ also satisfies the Properties (1)-(3). Therefore we only prove the property (4)

Proof

It is easy to see that $S_2(A, \mathbb{B})$ satisfies the properties (1)-(3). We only prove the 4th property.

As we obtain $d_p(A, \mathbb{O}) \geq d_p(A, \mathbb{B})$ and $d_p(A, \mathbb{O}) \geq d_p(\mathbb{B}, \mathbb{O})$ for $p > 0$ from Proposition 3.2 then,

$1 - d_p(A, \mathbb{B}) \geq 1 - d_p(A, \mathbb{O})$ and $1 - d_p(\mathbb{B}, \mathbb{O}) \geq 1 - d_p(A, \mathbb{O})$;

$1 + d_p(A, \mathbb{B}) \geq 1 + d_p(A, \mathbb{O})$ and $1 + d_p(\mathbb{B}, \mathbb{O}) \geq 1 + d_p(A, \mathbb{O})$.

Then the following inequalities,

$$\frac{1 - d_p(A, \mathbb{B})}{1 + d_p(A, \mathbb{B})} \geq \frac{1 - d_p(A, \mathbb{O})}{1 + d_p(A, \mathbb{O})}$$

$$\frac{1 - d_p(\mathbb{B}, \mathbb{O})}{1 + d_p(\mathbb{B}, \mathbb{O})} \geq \frac{1 - d_p(A, \mathbb{O})}{1 + d_p(A, \mathbb{O})}$$

Then, there are $S(A, \mathbb{O}) \leq S(A, \mathbb{B})$ and $S(A, \mathbb{O}) \leq S(\mathbb{B}, \mathbb{O})$. Hence the property (4) is satisfied. This completes the proof.

Example 3.2.2 Assume that we have the following three PNC-sets in a universe of distance $X = \{x_1, x_2\}$.

$$\begin{aligned} A_{PNCS} &= \{ \langle x_1, ([0.3, 0.5], [0.6, 0.8], [0.4, 0.6]), (0.4, 0.7, 0.5) \rangle, \\ &\quad \langle x_2, ([0.5, 0.6], [0.1, 0.3], [0.05, 0.2]), (0.55, 0.2, 0.1) \rangle \} \\ B_{PNCS} &= \{ \langle x_1, ([0.4, 0.6], [0.5, 0.7], [0.3, 0.5]), (0.5, 0.6, 0.4) \rangle, \\ &\quad \langle x_2, ([0.6, 0.7], [0.05, 0.2], [0.05, 0.1]), (0.65, 0.1, 0.1) \rangle \} \\ C_{PNCS} &= \{ \langle x_1, ([0.5, 0.7], [0.4, 0.6], [0.2, 0.4]), (0.6, 0.5, 0.3) \rangle, \\ &\quad \langle x_2, ([0.7, 0.8], [0.05, 0.1], [0.01, 0.1]), (0.75, 0.1, 0.05) \rangle \} \end{aligned}$$

Let the weight vector $w_{PNCS} = \{0.4, 0.6\}$. By applying (4), the similarity measures between the PNCS are as follows:

$S_1(A_{PNCS}, B_{PNCS}) = 0.9119,$

$S_1(B_{PNCS}, C_{PNCS}) = 0.9101,$

$S_1(A_{PNCS}, C_{PNCS}) = 0.8240$

Hence, $S_1(A_{PNCS}, C_{PNCS}) < S_1(A_{PNCS}, B_{PNCS})$ and $S_1(A_{PNCS}, C_{PNCS}) < S_1(B_{PNCS}, C_{PNCS})$

By applying (5), the similarity measure between PNCS are as follows

$S_2(A_{PNCS}, B_{PNCS}) = 0.8380,$

$S_2(B_{PNCS}, C_{PNCS}) = 0.8380,$

$S_2(A_{PNCS}, C_{PNCS}) = 0.7007$

Hence, $S_2(A_{PNCS}, C_{PNCS}) < S_2(A_{PNCS}, B_{PNCS})$ and $S_2(A_{PNCS}, C_{PNCS}) < S_2(B_{PNCS}, C_{PNCS})$

4. Application of Similarity measure on MADM Problem for air conditioning mechanism

4.1 An Algorithm based on Similarity measure on MADM Problem

Let $A = \{A_1, A_2, \dots, A_n\}$ be a set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be set of criteria. Assume the weight of the criterion entered by the decision makers $w_j, w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$.

Step 1: To begin the problem, we need to create a Pythagorean Neutrosophic Cubic decision matrix $\mathbb{D} = (\alpha_{ij})_{m \times n}$. In MADM environments, the concept of ideal point has been used to identify the best alternative in the decision set.

Step 2: We define an ideal PNCS for a benefit criterion in the ideal alternative A^* as

$$A^* = \{ \langle x, [T_A^-(x), T_A^+(x)], [I_A^-(x), I_A^+(x)], [F_A^-(x), F_A^+(x)], (\lambda_T^*(x), \lambda_I^*(x), \lambda_F^*(x)) \rangle \}$$

Step 3: Then Applying Equation (4) and Equation (5) we can get the similarity measure of $S_1(A^*, A_i)$ and $S_2(A^*, A_i)$ where $i = 1, 2, \dots, m$ between each alternative and the ideal alternative.

Step 4: Ranking order of all alternatives can be determined and the best one can be easily identified as well.

4.2 Illustrative Example

Let us consider the decision-making problem. Let N_1, N_2, N_3 be the set of customers who willing to purchase AC for their home. Let A_1, A_2, A_3, A_4 be the different brands of AC. Let C_1, C_2, C_3 be the attributes of the different brands of AC where C_1 denotes Cooling Capacity, C_2 denotes Cost Efficiency and C_3 denotes the Energy efficiency.

The weight vector of the criteria is given by $w = (0.40, 0.35, 0.25)$. The four possible alternatives are to be evaluated under the above three criteria by the form of PNCSs, as shown in the following Pythagorean Neutrosophic Cubic Decision matrix in the table representation:

	C_1 Cooling Capacity	C_2 Cost Efficiency	C_3 Energy Efficiency
A_1	[0.7, 0.8], [0.01, 0.1], [0.1, 0.2] (0.7, 0.1, 0.15)	[0.6, 0.7], [0.1, 0.2], [0.1, 0.3] (0.6, 0.15, 0.2)	[0.6, 0.7], [0.3, 0.4], [0.8, 0.9] (0.6, 0.4, 0.9)
A_2	[0.3, 0.6], [0.2, 0.3], [0.3, 0.4] (0.3, 0.2, 0.4)	[0.5, 0.6], [0.2, 0.3], [0.3, 0.4] (0.6, 0.3, 0.4)	[0.4, 0.5], [0.2, 0.4], [0.7, 0.9] (0.4, 0.2, 0.8)
A_3	[0.6, 0.7], [0.1, 0.2], [0.2, 0.3] (0.6, 0.1, 0.2)	[0.6, 0.7], [0.1, 0.2], [0.2, 0.3] (0.7, 0.1, 0.2)	[0.3, 0.6], [0.3, 0.5], [0.8, 0.9] (0.3, 0.4, 0.8)
A_4	[0.4, 0.5], [0.2, 0.3], [0.3, 0.4] (0.4, 0.2, 0.3)	[0.4, 0.6], [0.1, 0.3], [0.2, 0.4] (0.4, 0.1, 0.2)	[0.7, 0.9], [0.2, 0.3], [0.4, 0.5] (0.8, 0.2, 0.5)

Table – 1 : Relation between the brand and the criteria

From the Pythagorean Neutrosophic Cubic decision matrix \mathbb{D} we can obtain the following ideal alternative:

$$A^* = \left\{ \langle [0.3, 0.4], [0.3, 0.5], [0.8, 0.9], (0.3, 0.5, 0.8) \rangle, \langle [0.7, 0.8], [0.01, 0.1], [0.1, 0.2], (0.7, 0.1, 0.2) \rangle, \langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3], (0.6, 0.1, 0.3) \rangle \right\}$$

Then by using Equation (4), we obtain the following similarity measures of $S_1(A^*, A_i) (i = 1, 2, 3, 4)$:

$$\begin{aligned} S_1(A^*, A_1) &= 0.6336 \\ S_1(A^*, A_2) &= 0.7020 \\ S_1(A^*, A_3) &= 0.6635 \\ S_1(A^*, A_4) &= 0.7360 \end{aligned}$$

Therefore, the ranking order of the four alternatives is, $A_1 < A_3 < A_2 < A_4$. Amongst them A_4 is the best brand of AC.

Now by applying Equation (5), we can give the similarity measure of $S_2(A^*, A_i) (i = 1, 2, 3, 4)$ as follows:

$$\begin{aligned} S_2(A^*, A_1) &= 0.4637 \\ S_2(A^*, A_2) &= 0.5408 \\ S_2(A^*, A_3) &= 0.4964 \\ S_2(A^*, A_4) &= 0.5823 \end{aligned}$$

Thus, the ranking order of four alternatives is $A_1 < A_3 < A_2 < A_4$.

Therefore, we can conclude that using the two similarity measures the best brand to buy AC will be A_4 . i.e., A_4 brand AC will be the best one among the four.

5. Conclusion

We suggested the Pythagorean Neutrosophic Cubic Similarity Measure in this work along with clarifications for the Generalized Pythagorean Neutrosophic Cubic Weighted Distance Measure, Pythagorean Neutrosophic Cubic Weighted Hamming Distance Measure, and Pythagorean Neutrosophic Cubic Weighted Euclidean Distance Measure. After that, several of its highlights were examined. A MADM has been established utilizing Pythagorean Neutrosophic Cubic Set in the Pythagorean Neutrosophic Cubic Similarity Measure's application component. All options are categorized in order of preference using the Pythagorean Neutrosophic Cubic Similarity Measure, which also makes it simple to identify the best option. Lastly, a demonstration example has been provided.

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