

Inventory Analysis of Decaying Stock Under Demand Influenced by Supply and Time-Variable Storage Cost

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Abstract

In this study we have considered an economic order quantity model (EOQ) for deteriorating items under varying holding cost by assuming that the demand rate is stock dependent. In this paper we have considered time varying holding cost is linear function of time and shortage are not allowed. Products such as food grains, clothing and electronic devices have limited life span which diminishes as the season comes to a close. This chapter deals with the demand increases with the available inventory level, reflecting consumer behavior patterns commonly seen in retail and marketing contexts. Optimal solution is obtained with the help of differential calculus and optimality condition. The model is validating the optimal solution through sensitivity analysis and Numerical examples. It is also shown that the total cost function is convex. Informed decision-making approaches for inventory system facing real life complexities.

Keywords : *EOQ model, inventory, demand, deterioration, stock-dependent, varying holding cost,*

1. Introduction

Efficient inventory control is essential for enhancing supply chain performance, particularly when dealing with goods that have a short usable lifespan—such as perishable foods, seasonal apparel, and electronic gadgets. These products tend to lose value or functionality over time due to deterioration. Additionally, customer demand for such items is often influenced by the amount of inventory on display, a pattern commonly observed in retail and marketing environments.

Conventional Economic Order Quantity (EOQ) models typically operate under the assumption of constant demand and fixed holding costs. However, in practical scenarios, holding costs may vary with time as a result of factors like inflation, rising storage expenses, or resource constraints. In response to these real-world challenges, this study introduces an EOQ model.

Soni and Shah [1] determine the best ordering policies for retailers when demand has a constant component and is also affected by the stock. Saha *et al.* [2] investigate an EOQ models for the stock-dependent demand incorporating imprecise restriction. Mandal and Maiti [3-4] explain an inventory model for a single breakable item, where both the demand and the rate of breakage are functions (linear or non-linear) of the present stock. Giri *et al.* [5] developed a model where the unit production cost varies as a convex function of the production rate, which serves as the decision variable. Goyal and Chang [6] presented an ordering and transfer inventory model to find the ideal order quantity and frequency of transfers from storage to the display area for a retailer. Min *et al.* [7] developed a lot-sizing model for perishable goods (spoilage items) that includes current-stock dependent demand and trade credit arrangements. Liao *et al.* [8] studied an EOQ models for deteriorating items that require two storage facilities, linking the availability of trade credit to the size of the order. Chakrabarty and Chaudhuri [9] expressed as the joint decision of setting the optimal price and lot size when delayed payments are allowed for customers. Goyal and Giri [10] presented an reviewed and updated the existing literature on deteriorating inventory lot size models. Khanra *et al.* [11] developed an EOQ model over a finite time period for an item with a quadratic, time-dependent demand, explicitly including the possibility of shortages. Sarkar [12] provided an EOQ model for scenarios involving delayed payments and stock-dependent demand within an imperfect production environment. Saha *et al.* [13] established an EOQ models for items prone to damage, considering that customer interest is influenced by the quantity displayed, under conditions defined by approximate limitations. Dye *et al.* [14] presented an EOQ model for perishable goods featuring a selling rate based on current stock and partial backlogging that changes over time. Padmanabhan and Vrat [15] have studied EOQ models for perishable/deteriorating items where the selling rate is dependent on the stock level. Chung *et al.* [16] proposed a note on EOQ models for deteriorating items under stock dependent selling rate. Gupta and Vrat [17] proposed inventory models specifically addressing a consumption rate that is dependent on the stock level. Mandal and Phaujdar [18] developed a note on an inventory model with stock dependent consumption rate. You and Hsieh [19] proposed an EOQ model where demand is sensitive to both the current stock level and the price. Sana and Chaudhuri [20] have investigation on a volume-flexible inventory model that incorporates stock-dependent demand. Alfares [21] developed an inventory model that features both a demand rate dependent on the stock level and a variable holding cost. Dye [22] provided an inventory model that combines stock-dependent demand and partial backlogging under conditions allowing permissible delay in payments. Ghiami *et al.* [23] have studied a two-echelon inventory model for a deteriorating item that includes stock-dependent demand, partial backlogging, and capacity constraints. Tripathi [24] established deterministic inventory models that feature a non-linear holding cost which is independent of time but dependent on the stock level, while also considering a non-increasing demand rate that changes over time. Pandey [25] developed an EPQ model for deteriorating items by factoring in demand based on stock and a holding cost that changes over time. Halim and Ismail [26] analyzed an inventory model with overtime production for deteriorating items, where the demand is non-linearly dependent on both the selling price and the stock level. Yadav *et al.* [27] have investigation an inventory model that accounts for item deterioration, stock-dependent demand, a ramp-type demand pattern, reserve money, and carbon emissions. Singh [28] a production inventory model for deteriorating items considering the effects of holding cost, stock level, and selling price, assuming no backlogging. Datta and Pal [29] proposed strategy to boost demand for the current retail system, where the rate at which items are consumed is influenced by the available inventory. Palanivel and Uthayakumar [30] provided an inventory model that includes imperfect items, stock-dependent demand, and permissible delay in payments. Yadav *et al.* [31] established an inventory management optimization framework addresses the challenges of multiple, perishable products where demand exceeding supply leads to partially deferred orders. The model's economic calculations incorporate the cost of carbon emissions and are adjusted for the effects of inflation Singh [32] proposed an inventory model that integrates stock-dependent demand with different functions for holding cost. Teng [33] presented a model where both the demand rate and the holding cost rate are explicitly dependent on the current stock level. Ghosh *et al.* [34] provided an inventory model for a deteriorating item that requires two levels of storage and has stock-dependent demand. Datta and Paul [35] proposed an inventory system where the demand rate is sensitive to both the stock level and the price. Ouyang *et al.* [36] developed a model for deteriorating items with stock-dependent demand under the financial realities of inflation and the time-value of money. Chang *et al.* [37] proposed deteriorating items with stock-dependent demand and the time-value of money, specifically when a credit period is granted. Giri *et al.* [38] established a model for a deteriorating item with a stock-dependent demand rate. They also presented a model for damageable items featuring a variable replenishment rate, stock-dependent demand, and consideration of current on-hand units. Tripathi and Shah *et al.* [39] have studied the impact of greening efforts and deteriorating inventory policies for demand that is sensitive to both price and stock level.

2. Notations and Assumptions

2.1 Notations

$q(t)$: level of inventory at time 't'
$D(t)$: demand rate
c	: unit purchase cost
A	: ordering cost
θ	: constant deterioration rate $0 < \theta < 1$
q_0	: the economic order quantity
Q	: order quantity
h	: holding cost
T	: inventory stock cycle length
OC	: ordering cost
HC	: holding cost
DC	: deteriorating cost
TC	: Total cost
T^*	: optimal T

2.2. Assumptions

- (i) Demand rate is measured as the stock dependent *i.e.* $D(t) = a + bq(t)$; $a > 0$, $b > 0$.
- (ii) Holding cost is time dependent where $h(t) = h + \alpha t$, $h > 0$ and $\alpha > 0$.
- (iii) deterioration rate is considered as constant.
- (iv) Shortage is not allowed.
- (v) Lead time is zero.

3. Mathematical Formulation

We examine a deterministic inventory model defined over a limited planning period, in which the inventory is subject to deterioration and demand that depends on the current stock level, while the holding cost changes over time. Let the inventory level at any given moment be represented by $q(t)$.

$$\frac{dq(t)}{dt} + \theta q(t) = -[a + bq(t)]; 0 \leq t \leq T \tag{1}$$

$$\text{With the conditions } q(t) = 0 \tag{2}$$

Solution of (1) under the condition (2) is:

$$q(t) = \frac{a}{(b + \theta)} \left\{ e^{-(b+\theta)t} - 1 \right\}; 0 \leq t \leq T \tag{3}$$

Thus, the initial order quantity is obtained by putting the boundary condition $q(0) = q_0$ in Eqⁿ (2).

$$q_0 = q(0) = \frac{a}{b + \theta} \left\{ e^{-(b+\theta)0} - 1 \right\} \tag{4}$$

The profit function per cycle are contain below

(i) The ordering cost is $(OC) = A$

(ii) The total demand during the cycle period $[0, T]$ is

$$\int_0^T D(t) dt = \int_0^T \{a + bq(t)\} dt$$

$$= \frac{a}{(b+\theta)}(1-b)e^{-(b+\theta)T} - aT - \frac{a}{(b+\theta)} \left\{ 1 + \frac{b}{(b+\theta)} - bT \right\} \quad (5)$$

(iii) The deterioration cost (DC) for the cycle $[0, T]$

$$= C_d \times (\text{the no of deteriorated units})$$

$$= C_d \frac{a}{(b+\theta)} \left\{ (1-b)e^{-(b+\theta)T} - a(b+\theta)T - \left(1 + \frac{b}{b+\theta} - bT \right) \right\} \quad (6)$$

(iv) The total inventory holding cost for cycle $[0, T]$

$$= \int_0^T (h + \alpha t) q(t) dt$$

$$= \frac{a}{(b+\theta)^3} \left\{ \alpha + h(b+\theta) \right\} \left\{ 1 - e^{-(b+\theta)T} \right\} - \frac{aT}{2(b+\theta)} (2h + \alpha T) - \frac{a\alpha T e^{-(b+\theta)T}}{(b+\theta)^2} \quad (7)$$

Total variable cost $TC = OC + DC + HC$

$$TC = \frac{A_0}{T} + \frac{a}{(b+\theta)^2} \left\{ h + bC_d(b+\theta) + \frac{\alpha}{(b+\theta)} \right\} \left\{ 1 - e^{-(b+\theta)T} \right\} - C_d a^2 b T$$

$$+ \frac{C_d a^2 b^2}{(b+\theta)^2} \left\{ e^{-(b+\theta)T} - T - 1 \right\} - \frac{aT}{2(b+\theta)} (2h + \alpha T) - \frac{a\alpha T e^{-(b+\theta)T}}{(b+\theta)^2} \quad (8)$$

Optimality Condition

The twice differentiation of Eq.(8) w.r.t. 'T' are:

$$\frac{dTC}{dT} = -\frac{A_0}{T^2} - \frac{a}{(b+\theta)} \left\{ h + bC_d(b+\theta) + \frac{\alpha}{(b+\theta)} \right\} e^{-(b+\theta)T} - C_d a^2 b$$

$$+ \frac{C_d a^2 b^2}{(b+\theta)^2} \left\{ (b+\theta) e^{-(b+\theta)T} - 1 \right\} - \frac{1}{(b+\theta)} \left\{ a + \alpha a T + a\alpha T e^{-(b+\theta)T} + \frac{a\alpha}{(b+\theta)} e^{-(b+\theta)T} \right\}$$

$$\frac{d^2TC}{dT^2} = \frac{2A_0}{T^3} - a e^{-(b+\theta)T} \left\{ h + bC_d(b+\theta) + \frac{\alpha}{(b+\theta)} \right\} - C_d a^2 b^2 e^{-(b+\theta)T} - \frac{1}{(b+\theta)} \left\{ \alpha(a+1) \right.$$

$$\left. + a\alpha e^{-(b+\theta)T} (1 - T(b+\theta)) \right\} \quad (\text{i.e. } TC^* \text{ is minimum})$$

and

The condition of minimization is also shown by the following graph:

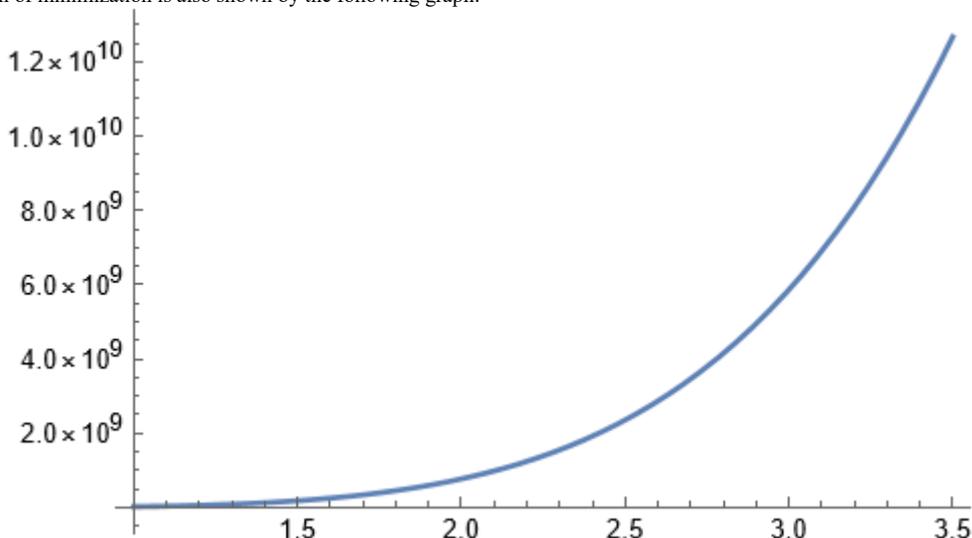


Fig. 1. Graph between T (x-axis, 0.0 – 0.30) and TC (y-axis)

T^* is obtained by solving $\frac{d(TC)}{dT} = 0$

$$-A_0(b+\theta)^2 + aT^2(b+\theta)e^{-(b+\theta)T} \left\{ h + bC_d(b+\theta) + \frac{\alpha}{b+\theta} \right\} - C_d a^2 b T^2 (b+\theta)^2 - C_d a^2 b^2 T^2 \left\{ 1 - (b+\theta)e^{-(b+\theta)T} \right\} - aT^2(b+\theta)(1+\alpha T) + \alpha a T^2 e^{-(b+\theta)T} \left\{ 1 + (b+\theta)T^2 \right\} = 0 \quad (9)$$

4. Numerical Example

Let us consider the cost parameters, $A = 100, a = 50, b = 10, C_d = 1, \alpha = 0.4, h = 0.7, \theta = 0.5$. Putting these values in Eq. (9). This gives $T^* = 0.408439$ yrs, corresponding $Q^* = 23.3689, HC^* = 1.77378, DC^* = 224.613$, and $TC^* = 326.39$

Sensitivity Analysis

This examination is demonstrated in Table 1, where, starting from the values defined in the Numerical Example, we keep every parameter constant and systematically perturb one constraint to gauge its effect.

Table 1: The effect of parameter on T^*, Q^*, HC^*, DC^* , and TC^*

Parameters		Optimal value				
		T^*	Q^*	HC^*	DC^*	TC^*
A	110	0.407936	23.2863	1.76245	198.149	309.91
	120	0.407430	23.2033	1.75110	171.592	293.34
	130	0.406922	23.1201	1.73975	144.996	276.74
	140	0.406411	23.0365	1.72837	118.309	260.04
	150	0.405897	22.9526	1.71697	91.5327	243.25
a	60	0.409159	28.1850	2.14808	321.783	423.93
	70	0.409497	32.9605	2.51684	417.914	520.43
	80	0.409657	37.7114	2.88222	513.476	616.36
	90	0.409727	42.4461	3.24536	608.690	711.94
	100	0.409748	47.1692	3.60692	703.450	807.06
b	20	0.1993800	10.4041	0.307276	230.513	330.82
	30	0.1304910	6.45922	0.112005	234.685	334.80
	40	0.0961397	4.55139	0.053792	240.183	340.24
	50	0.0754997	3.42152	0.029625	248.916	348.95
	60	0.0616721	2.66911	0.017576	255.813	355.99
α	0.5	0.408442	23.3694	1.90380	224.771	326.68
	0.6	0.408445	23.3699	2.03383	224.929	326.97
	0.7	0.408447	23.3702	2.16384	225.034	327.20
	0.8	0.408450	23.3707	2.29389	225.192	327.49
	0.9	0.408953	23.3712	2.42395	225.350	327.77
h	0.8	0.408444	23.3698	1.95305	224.771	326.83
	1.0	0.408453	23.3612	2.31162	224.929	327.66
	1.2	0.408462	23.3727	2.67027	225.034	328.49
	1.4	0.408471	23.3742	3.02901	225.192	329.33
	1.6	0.408480	23.3757	3.38784	225.350	330.16
d	2	0.410897	23.7747	1.82975	709.728	811.560
	3	0.411703	23.9084	1.84832	1193.73	1295.58
	4	0.412104	23.9751	1.85761	1677.55	1779.41
	5	0.412343	24.0149	1.86315	2161.04	2262.90
	6	0.412503	24.0415	1.86687	2644.80	2746.67
θ	0.6	0.406287	23.4290	1.77382	224.530	326.30
	0.7	0.404177	23.4897	1.77397	224.405	326.18
	0.8	0.402110	23.5515	1.77427	224.358	326.13
	0.9	0.400084	23.6141	1.77470	224.347	326.12
	1.0	0.398097	23.6775	1.77524	224.329	326.10

From the above Table 1 following inferences can be summarized.

- (i) On increasing a, α, h, d ; T^*, Q^*, HC^*, DC^* and TC^* are increasing
- (ii) On increasing A ; T^*, Q^*, HC^*, DC^* and TC^* are decreasing.
- (iii) On increasing b , T^*, Q^*, HC^*, DC^* are decreasing while DC^* and TC^* are rising.
- (iv) On increasing θ , T^*, DC^* and TC^* are decreasing while Q^* and HC^* are increasing.

Conclusion and Future Research

This study introduces an enhanced Economic Order Quantity (EOQ) model that integrates three practical aspects frequently omitted in traditional models: inventory-level-dependent demand, holding costs that vary over time, and the natural deterioration of items. The mathematical formulation illustrates how these components collectively impact the determination of an optimal inventory strategy. The framework offers flexibility for future development, such as incorporating shortages, partial backorders, inflationary effects, or stochastic deterioration patterns. It provides a valuable decision-making tool for supply chain professionals managing perishable or seasonal products with fluctuating demand and cost

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