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Lattice Valued Picture Fuzzy Mappings

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ABSTRACT:
In this paper, we introduce Lattice Valued Picture Fuzzy Mappings and inverse Lattice Valued Picture Fuzzy Mappings. The properties of Lattice Valued Picture Fuzzy Mappings are discussed with illustrations.

Keywords: Lattice Valued Picture Fuzzy Mapping, Inverse Lattice Valued Picture Fuzzy Mapping, composition of Lattice Valued Picture Fuzzy Mapping
AMS Subject Classification: 03E72.

1. INTRODUCTION

In the recent years, the field of fuzzy set theory has evolved to better capture the complexities of real-world decision-making processes. Fuzzy sets introduced by Zadeh (*cf.* [9]) could not model uncertainty. To overcome this, Intuitionistic fuzzy sets was defined by Atanassov (*cf.* [1]). Smarandache (*cf.* [6]) proposed the concept of Neutrosophic sets. Jose James and Sunil C. Mathew (*cf.* [4]) introduced Lattice valued Neutrosophic sets and defined Lattice valued Neutrosophic mappings. Picture fuzzy set, introduced by B. C. Cuong (*cf.* [2]), introduces a novel approach to represent and analyze fuzzy sets with higher degrees of granularity. Sijia Zhu studied the distance measures of picture fuzzy sets (*cf.* [12]) and interval-valued picture fuzzy sets with their applications (*cf.* [11]). New distance and similarity measures of picture fuzzy sets were defined by Henan Li (*cf.* [5]). Chen Zhang and Zengtai Gong (*cf.* [10]) introduced Picture Fuzzy concept to Lattice models for Knowledge structure analysis. Tweena Evangelin P and A. Francina Shalini defined Lattice Valued Picture Fuzzy Sets (*cf.* [7]) by inducing picture fuzzy sets to the theory of lattices and developed their topologies (*cf.* [8]). Building upon the concept of Lattice Valued Picture Fuzzy Sets, Lattice Valued Picture Fuzzy Mapping is used to model relationships, particularly when there are multiple levels of uncertainty involved. Lattice Valued Picture Fuzzy Mapping holds considerable promise in various research and application areas.

In this paper, we define Lattice Valued Picture Fuzzy Mappings and inverse Lattice Valued Picture Fuzzy Mappings. Some of their properties are investigated.

1.1 Preliminaries

Definition 1.1. *(cf. [3])* Let P be a non-empty ordered set. If $x \vee y$ and $x \wedge y$ exist for all $x, y \in P$, then P is called a lattice.

Definition 1.2. *(cf. [2])* A picture fuzzy set $A = \{\langle x, \mu_A(x), \eta_A(x), \nu_A(x)\rangle | x \in X\}$ where $\mu_A(x), \eta_A(x), \nu_A(x) \in [0,1]$ are called the positive, neutral and negative membership degrees of x in A and satisfy the condition: $\forall x \in X, \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1$. Now, $1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))$ is the degree of refusal membership of $x \in X$.

Definition 1.3. *(cf. [7])* Consider the universe X and a nontrivial complete boolean lattice L. A Lattice Valued Picture Fuzzy Set (LVPFS) $A_L = \{ \langle x, \mu_{(A_L)}(x), \eta_{(A_L)}(x), \nu_{(A_L)}(x) \rangle \mid x \in X \}$ where $\mu_{(A_L)}(x) : X \to L$ is the positive membership degree, $\eta_{(A_L)}(x) : X \to L$ is the neutral membership degree and $\nu_{(A_L)}(x) : X \to L$ is the negative membership degree.

$\pi_{(A_L)}(x) = (\mu_{(A_L)}(x) \vee \eta_{(A_L)}(x) \vee \nu_{(A_L)}(x))'$ is the refusal membership degree of x in A_L.

LVPFS(X) represents set of Lattice Valued Picture Fuzzy Sets in *X*.

Definition 1.4. *(cf. [7])*

1. $A_{1_L} \subseteq A_{2_L} \Leftrightarrow \mu_{(A_{1_L})}(x) \leq \mu_{(A_{2_L})}(x), \eta_{(A_{1_L})}(x) \leq \eta_{(A_{2_L})}(x), \nu_{(A_{1_L})}(x) \geq \nu_{(A_{2_L})}(x)$

2. $A_{1_L} \cup A_{2_L} = \left\{ \left\langle x, \mu_{(A_{1_L} \cup A_{2_L})}(x), \eta_{(A_{1_L} \cup A_{2_L})}(x), \nu_{(A_{1_L} \cup A_{2_L})}(x) \right\rangle \middle| x \in X \right\}$, *where*

$\mu_{(A_{1_L} \cup A_{2_L})}(x) = \mu_{(A_{1_L})}(x) \vee \mu_{(A_{2_L})}(x)$,
$\eta_{(A_{1_L} \cup A_{2_L})}(x) = \eta_{(A_{1_L})}(x) \wedge \eta_{(A_{2_L})}(x)$,
$\nu_{(A_{1_L} \cup A_{2_L})}(x) = \nu_{(A_{1_L})}(x) \wedge \nu_{(A_{2_L})}(x)$.

3. $A_{1_L} \cap A_{2_L} = \left\{ \left\langle x, \mu_{(A_{1_L} \cap A_{2_L})}(x), \eta_{(A_{1_L} \cap A_{2_L})}(x), \nu_{(A_{1_L} \cap A_{2_L})}(x) \right\rangle \middle| x \in X \right\}$, *where*

$\mu_{(A_{1_L} \cap A_{2_L})}(x) = \mu_{(A_{1_L})}(x) \wedge \mu_{(A_{2_L})}(x)$,
$\eta_{(A_{1_L} \cap A_{2_L})}(x) = \eta_{(A_{1_L})}(x) \wedge \eta_{(A_{2_L})}(x)$,
$\nu_{(A_{1_L} \cap A_{2_L})}(x) = \nu_{(A_{1_L})}(x) \vee \nu_{(A_{2_L})}(x)$.

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2. MAIN RESULTS

Definition 2.1. Let $A_L \in LVPFS(X), B_L \in LVPFS(Y)$ and Φ be a mapping from X to Y. The Lattice Valued Picture Fuzzy Mapping *(LVPFM)* from *LVPFS(X)* to *LVPFS(Y)* induced by Φ is defined as

$$\vec{\Phi}(A_L) = \{ \langle y, \mu_{(\vec{\Phi}(A_L))}(y), \eta_{(\vec{\Phi}(A_L))}(y), \nu_{(\vec{\Phi}(A_L))}(y) \rangle \mid y \in Y \}$$

where

$$\mu_{(\vec{\Phi}(A_L))}(y) = \bigvee_{x \in \Phi^{-1}(y)} \{ \mu_{(A_L)}(x) \},$$

$$\eta_{(\vec{\Phi}(A_L))}(y) = \bigwedge_{x \in \Phi^{-1}(y)} \{ \eta_{(A_L)}(x) \},$$

$$\nu_{(\vec{\Phi}(A_L))}(y) = \bigwedge_{x \in \Phi^{-1}(y)} \{ \nu_{(A_L)}(x) \}.$$

$\pi_{(\vec{\Phi}(A_L))}(y) = \left(\mu_{(\vec{\Phi}(A_L))}(y) \vee \eta_{(\vec{\Phi}(A_L))}(y) \vee \nu_{(\vec{\Phi}(A_L))}(y) \right)'$ is the refusal degree of y in $\vec{\Phi}(A_L)$.

Example 2.2. *Let* $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2\}$ and Φ be a mapping such that $\Phi(x_1) = y_1, \Phi(x_2) = y_2, \Phi(x_3) = y_1$.

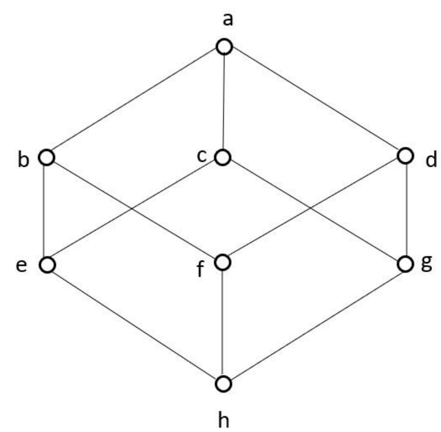

Figure 1: Boolean lattice of order 2

Let $A_L = \{\langle x_1, c, f, g\rangle, \langle x_2, a, d, b\rangle, \langle x_3, e, g, c\rangle\}$ and $B_L = \{\langle y_1, d, a, c\rangle, \langle y_2, g, e, b\rangle\}$ be *LVPFSs* in X and Y respectively, where L is the lattice in *figure 1.*
Then, the *LVPFM* from A_L to B_L is $\vec{\Phi} : A_L \to B_L$ such that $\vec{\Phi}(A_L) = \{\langle y_1, a, h, g\rangle, \langle y_2, a, d, b\rangle\}$.

Definition 2.3. Let $A_L \in LVPFS(X), B_L \in LVPFS(Y)$ and Φ be a mapping from X to Y. The inverse Lattice Valued Picture Fuzzy Mapping from *LVPFS*(Y) to *LVPFS*(X) induced by Φ is defined as

$$\overleftarrow{\Phi}(B_L) = \{ \langle x, \mu_{(\overleftarrow{\Phi}(B_L))}(x), \eta_{(\overleftarrow{\Phi}(B_L))}(x), \nu_{(\overleftarrow{\Phi}(B_L))}(x) \rangle \mid x \in X \}$$

where

$$\mu_{(\overleftarrow{\Phi}(B_L))}(x) = \mu_{(B_L)}(\Phi(x)),$$

$$\eta_{(\overleftarrow{\Phi}(B_L))}(x) = \eta_{(B_L)}(\Phi(x)),$$

$$\nu_{(\overleftarrow{\Phi}(B_L))}(x) = \nu_{(B_L)}(\Phi(x)).$$

$\pi_{(\overleftarrow{\Phi}(B_L))}(y) = \left(\mu_{(\overleftarrow{\Phi}(B_L))}(x) \vee \eta_{(\overleftarrow{\Phi}(B_L))}(x) \vee \nu_{(\overleftarrow{\Phi}(B_L))}(x) \right)'$ is the refusal degree of x in $\overleftarrow{\Phi}(B_L)$.

Example 2.4. Consider the LVPFM in example 2.2.

The inverse LVPFM is $\overleftarrow{\Phi}(B_L) = \{\langle x_1, d, a, c\rangle, \langle x_2, g, e, b\rangle, \langle x_3, d, a, c\rangle\}$.

Theorem 2.5. Let $\Phi : X \to Y$ and $\Phi : LVPFS(X) \to LVPFS(Y)$ be a LVPF mapping. Then,

1. $\overrightarrow{\Phi}$ is one-to-one iff Φ is one-to-one
2. $\overrightarrow{\Phi}$ is onto iff Φ is onto
3. $\overrightarrow{\Phi}$ is bijective iff Φ is bijective

Proof. The proof is obvious.

Theorem 2.6. $\overrightarrow{\Phi}$ is injective if $\overrightarrow{\Phi}\ o\ \overleftarrow{\Phi} = I$ where I is the identity mapping on $LVPFS(X)$.

Proof. Let $A_L \in LVPFS(X)$ and suppose that $\overrightarrow{\Phi}$ is injective. Then,

$$\mu_{(\overrightarrow{\Phi}(A_L))}(y) = \bigvee_{x \in \Phi^{-1}(y)} \{\mu_{(A_L)}(x)\} = \mu_{(A_L)}(x),$$

$$\eta_{(\overrightarrow{\Phi}(A_L))}(y) = \bigwedge_{x \in \Phi^{-1}(y)} \{\eta_{(A_L)}(x)\} = \eta_{(A_L)}(x),$$

$$\nu_{(\overrightarrow{\Phi}(A_L))}(y) = \bigwedge_{x \in \Phi^{-1}(y)} \{\nu_{(A_L)}(x)\} = \nu_{(A_L)}(x).$$

$$\Rightarrow \overrightarrow{\Phi}(A_L) = \left\{\left\langle y, \mu_{(\overrightarrow{\Phi}(A_L))}(y), \eta_{(\overrightarrow{\Phi}(A_L))}(y), \nu_{(\overrightarrow{\Phi}(A_L))}(y)\right\rangle \mid y \in Y\right\}$$

$$\begin{aligned}(\overrightarrow{\Phi}\ o\ \overleftarrow{\Phi})(A_L) &= \overleftarrow{\Phi}\left(\overrightarrow{\Phi}(A_L)\right)\\ &= \left\{\left\langle x, \mu_{(\overrightarrow{\Phi}(A_L))}(\Phi(x)), \eta_{(\overrightarrow{\Phi}(A_L))}(\Phi(x)), \nu_{(\overrightarrow{\Phi}(A_L))}(\Phi(x))\right\rangle \mid x \in X\right\}\\ &= \left\{\left\langle x, \mu_{(\overrightarrow{\Phi}(A_L))}(y), \eta_{(\overrightarrow{\Phi}(A_L))}(y), \nu_{(\overrightarrow{\Phi}(A_L))}(y)\right\rangle \mid x \in X\right\}\\ &= \left\{\left\langle x, \mu_{(A_L)}(x), \eta_{(A_L)}(x), \nu_{(A_L)}(x)\right\rangle \mid x \in X\right\}\\ &= A_L\end{aligned}$$

$$\Rightarrow \overrightarrow{\Phi}\ o\ \overleftarrow{\Phi} = I$$

Conversely, if $\overrightarrow{\Phi}\ o\ \overleftarrow{\Phi} = I$, then $\overrightarrow{\Phi}$ is injective.

Theorem 2.7. Let $A_L \in LVPFS(X), B_L \in LVPFS(Y)$ and Φ be a mapping from X to Y. Then,

1. $\overleftarrow{\Phi}\left(\overrightarrow{\Phi}(A_L)\right) = A_L$ if Φ is injective
2. $\overrightarrow{\Phi}\left(\overleftarrow{\Phi}(B_L)\right) = B_L$ if Φ is surjective

Proof.

1. The proof is a consequence of Theorem 2.6.
2. Let Φ be surjective.

$$\overleftarrow{\Phi}(B_L) = \left\{\left\langle x, \mu_{(B_L)}(\Phi(x)), \eta_{(B_L)}(\Phi(x)), \nu_{(B_L)}(\Phi(x))\right\rangle \mid x \in X\right\}$$

$$\begin{aligned}\overrightarrow{\Phi}\left(\overleftarrow{\Phi}(B_L)\right) &= \left\{\left\langle y, \bigvee_{x \in \Phi^{-1}(y)} \mu_{(B_L)}(\Phi(x)), \bigwedge_{x \in \Phi^{-1}(y)} \eta_{(B_L)}(\Phi(x)), \bigwedge_{x \in \Phi^{-1}(y)} \nu_{(B_L)}(\Phi(x))\right\rangle \mid x \in X, y \in Y\right\}\\ &= \left\{\left\langle y, \mu_{(B_L)}(y), \eta_{(B_L)}(y), \nu_{(B_L)}(y)\right\rangle \mid y \in Y\right\}\\ &= B_L\end{aligned}$$

Theorem 2.8. Let $A_{1_L}, A_{2_L} \in LVPFS(X), B_{1_L}, B_{2_L} \in LVPFS(Y)$ and Φ be a mapping from X to Y. Then,

1. $A_{1_L} \subseteq A_{2_L} \Rightarrow \overrightarrow{\Phi}(A_{1_L}) \subseteq \overrightarrow{\Phi}(A_{2_L})$
2. $B_{1_L} \subseteq B_{2_L} \Rightarrow \overleftarrow{\Phi}(B_{1_L}) \subseteq \overleftarrow{\Phi}(B_{2_L})$

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Proof.

1. $A_{1_L} \subseteq A_{2_L} \Leftrightarrow \mu_{(A_{1_L})}(x) \leq \mu_{(A_{2_L})}(x), \eta_{(A_{1_L})}(x) \leq \eta_{(A_{2_L})}(x), \nu_{(A_{1_L})}(x) \geq \nu_{(A_{2_L})}(x)$

$$\Rightarrow \bigvee_{x\in \Phi^{-1}(y)}\left(\mu_{(A_{1_L})}(x)\right) \leq \bigvee_{x\in \Phi^{-1}(y)}\left(\mu_{(A_{2_L})}(x)\right)$$
$$\bigwedge_{x\in \Phi^{-1}(y)}\left(\eta_{(A_{1_L})}(x)\right) \leq \bigwedge_{x\in \Phi^{-1}(y)}\left(\eta_{(A_{2_L})}(x)\right)$$
$$\bigwedge_{x\in \Phi^{-1}(y)}\left(\nu_{(A_{1_L})}(x)\right) \geq \bigwedge_{x\in \Phi^{-1}(y)}\left(\nu_{(A_{2_L})}(x)\right)$$

$$\Rightarrow \mu_{(\vec{\Phi}(A_{1_L}))}(y) \leq \mu_{(\vec{\Phi}(A_{2_L}))}(y),$$
$$\eta_{(\vec{\Phi}(A_{1_L}))}(y) \leq \eta_{(\vec{\Phi}(A_{2_L}))}(y),$$
$$\nu_{(\vec{\Phi}(A_{1_L}))}(y) \geq \nu_{(\vec{\Phi}(A_{2_L}))}(y)$$

$$\Rightarrow \vec{\Phi}(A_{1_L}) \subseteq \vec{\Phi}(A_{2_L})$$

2. $B_{1_L} \subseteq B_{2_L} \Leftrightarrow \mu_{(B_{1_L})}(x) \leq \mu_{(B_{2_L})}(x), \eta_{(B_{1_L})}(x) \leq \eta_{(B_{2_L})}(x), \nu_{(B_{1_L})}(x) \geq \nu_{(B_{2_L})}(x)$

$$\Rightarrow \mu_{(B_{1_L})}(\Phi(x)) \leq \mu_{(B_{2_L})}(\Phi(x)),$$
$$\eta_{(B_{1_L})}(\Phi(x)) \leq \eta_{(B_{2_L})}(\Phi(x)),$$
$$\nu_{(B_{1_L})}(\Phi(x)) \geq \nu_{(B_{2_L})}(\Phi(x))$$

$$\Rightarrow \mu_{(\overleftarrow{\Phi}(B_{1_L}))}(x) \leq \mu_{(\overleftarrow{\Phi}(B_{1_L}))}(x),$$
$$\eta_{(\overleftarrow{\Phi}(B_{1_L}))}(x) \leq \eta_{(\overleftarrow{\Phi}(B_{1_L}))}(x),$$
$$\nu_{(\overleftarrow{\Phi}(B_{1_L}))}(x) \geq \nu_{(\overleftarrow{\Phi}(B_{1_L}))}(x)$$

$$\Rightarrow \overleftarrow{\Phi}(B_{1_L}) \subseteq \overleftarrow{\Phi}(B_{2_L})$$

Theorem 2.9. Let $A_L, B_L \in LVPFS(X)$, $B_{1_L}, B_{2_L} \in LVPFS(Y)$ and Φ be a mapping from X to Y. Then,

1. $\vec{\Phi}\,(A_L \cup B_L) = \vec{\Phi}\,(A_L) \cup \vec{\Phi}\,(B_L)$
2. $\vec{\Phi}\,(A_L \cap B_L) = \vec{\Phi}\,(A_L) \cap \vec{\Phi}\,(B_L)$

Proof.

1. $\vec{\Phi}\,(A_L) = \left\{\left\langle y, \bigvee_{x\in \Phi^{-1}(y)}\{\mu_{(A_L)}(x)\}, \bigwedge_{x\in \Phi^{-1}(y)}\{\eta_{(A_L)}(x)\}, \bigwedge_{x\in \Phi^{-1}(y)}\{\nu_{(A_L)}(x)\}\right\rangle \mid y \in Y\right\}$

$$\vec{\Phi}\,(B_L) = \left\{\left\langle y, \bigvee_{x\in \Phi^{-1}(y)}\{\mu_{(B_L)}(x)\}, \bigwedge_{x\in \Phi^{-1}(y)}\{\eta_{(B_L)}(x)\}, \bigwedge_{x\in \Phi^{-1}(y)}\{\nu_{(B_L)}(x)\}\right\rangle \mid y \in Y\right\}$$

$$\mu_{(\vec{\Phi}\,(A_L)\cup\vec{\Phi}\,(B_L))}(y) = \left(\bigvee_{x\in \Phi^{-1}(y)}\{\mu_{(A_L)}(x)\}\right) \vee \left(\bigvee_{x\in \Phi^{-1}(y)}\{\mu_{(B_L)}(x)\}\right)$$
$$= \bigvee_{x\in \Phi^{-1}(y)}\left(\mu_{(A_L)}(x) \vee \mu_{(B_L)}(x)\right)$$
$$= \bigvee_{x\in \Phi^{-1}(y)}\left(\mu_{(A_L\cup B_L)}(x)\right)$$
$$= \mu_{(\vec{\Phi}\,(A_L\cup B_L))}(y)$$

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$$\eta_{\left(\vec{\Phi}(A_L)\cup\vec{\Phi}(B_L)\right)}(y) = \left(\bigwedge_{x\in \Phi^{-1}(y)}\{\eta_{(A_L)}(x)\}\right)\wedge\left(\bigwedge_{x\in \Phi^{-1}(y)}\{\eta_{(B_L)}(x)\}\right)$$

$$= \bigwedge_{x\in \Phi^{-1}(y)}\left(\eta_{(A_L)}(x)\wedge \eta_{(B_L)}(x)\right)$$

$$= \bigwedge_{x\in \Phi^{-1}(y)}\left(\eta_{(A_L\cup B_L)}(x)\right)$$

$$= \eta_{\left(\vec{\Phi}(A_L\cup B_L)\right)}(y)$$

$$\nu_{\left(\vec{\Phi}(A_L)\cup\vec{\Phi}(B_L)\right)}(y) = \left(\bigwedge_{x\in \Phi^{-1}(y)}\{\nu_{(A_L)}(x)\}\right)\wedge\left(\bigwedge_{x\in \Phi^{-1}(y)}\{\nu_{(B_L)}(x)\}\right)$$

$$= \bigwedge_{x\in \Phi^{-1}(y)}\left(\nu_{(A_L)}(x)\wedge \nu_{(B_L)}(x)\right)$$

$$= \bigwedge_{x\in \Phi^{-1}(y)}\left(\nu_{(A_L\cup B_L)}(x)\right)$$

$$= \nu_{\left(\vec{\Phi}(A_L\cup B_L)\right)}(y)$$

Thus, $\vec{\Phi}(A_L\cup B_L) = \vec{\Phi}(A_L)\cup\vec{\Phi}(B_L)$.

2. The proof is analogous to that of (1).

Theorem 2.10. Let $A_{1_L}, A_{2_L} \in LVPFS(X), A_L, B_L \in LVPFS(Y)$ and Φ be a mapping from X to Y. Then,

1. $\overleftarrow{\Phi}(A_L\cup B_L) = \overleftarrow{\Phi}(A_L)\cup\overleftarrow{\Phi}(B_L)$
2. $\overleftarrow{\Phi}(A_L\cap B_L) = \overleftarrow{\Phi}(A_L)\cap\overleftarrow{\Phi}(B_L)$

Proof.

1. $$\mu_{\left(\overleftarrow{\Phi}(A_L\cup B_L)\right)}(x) = \mu_{(A_L\cup B_L)}(\Phi(x))$$

$$= \left(\mu_{(A_L)}(\Phi(x))\right)\vee\left(\mu_{(B_L)}(\Phi(x))\right)$$

$$= \left(\mu_{\left(\overleftarrow{\Phi}(A_L)\right)}(x)\right)\vee\left(\mu_{\left(\overleftarrow{\Phi}(B_L)\right)}(x)\right)$$

$$= \mu_{\left(\overleftarrow{\Phi}(A_L)\cup\overleftarrow{\Phi}(B_L)\right)}(x)$$

$$\eta_{\left(\overleftarrow{\Phi}(A_L\cup B_L)\right)}(x) = \eta_{(A_L\cup B_L)}(\Phi(x))$$

$$= \left(\eta_{(A_L)}(\Phi(x))\right)\wedge\left(\eta_{(B_L)}(\Phi(x))\right)$$

$$= \left(\eta_{\left(\overleftarrow{\Phi}(A_L)\right)}(x)\right)\wedge\left(\eta_{\left(\overleftarrow{\Phi}(B_L)\right)}(x)\right)$$

$$= \eta_{\left(\overleftarrow{\Phi}(A_L)\cup\overleftarrow{\Phi}(B_L)\right)}(x)$$

$$\nu_{\left(\overleftarrow{\Phi}(A_L\cup B_L)\right)}(x) = \nu_{(A_L\cup B_L)}(\Phi(x))$$

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$$= \left(\nu_{(A_L)}(\Phi(x))\right) \wedge \left(\nu_{(B_L)}(\Phi(x))\right)$$

$$= \left(\nu_{\left(\overleftarrow{\Phi}(A_L)\right)}(x)\right) \wedge \left(\nu_{\left(\overleftarrow{\Phi}(B_L)\right)}(x)\right)$$

$$= \nu_{\left(\overleftarrow{\Phi}(A_L) \cup \overleftarrow{\Phi}(B_L)\right)}(x)$$

Thus, $\overleftarrow{\Phi}(A_L \cup B_L) = \overleftarrow{\Phi}(A_L) \cup \overleftarrow{\Phi}(B_L)$.

2. The proof is analogous to that of (1).

CONCLUSION

In this paper, we have defined Lattice Valued Picture Fuzzy Mappings and inverse Lattice Valued Picture Fuzzy Mappings. We have also investigated some of their properties. We can further probe the applications in real life situations.

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