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# Novel Operations on Pythagorean Neutrosophic Super Hypersoft Matrices: Applications in Uncertain Multi-Criteria Decision Making

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Abstract:
The Pythagorean Neutrosophic Super Hypersoft Matrix (PNSHM) Theory is the concept under investigation in this paper. Here, we offer several new notations (operations) such PNSHS-sub matrix, Equal PNSHM, Null PNSHM, Universal PNSHM, and Complement PNSHM. PNSHM is the matrix representation of a Pythagorean Neutrosophic Super Hypersoft Set (PNSHSS).

**Keywords:** Neutrosophic Soft Set, Neutrosophic Hypersoft set, Pythagorean Neutrosophic Hypersoft set, Pythagorean Neutrosophic Super Hypersoft set, Pythagorean Neutrosphic Super Hyper soft Matrix

## 1. INTRODUCTION

A. Zadeh established a foundation for fuzzy sets in 1965 [14]. The level of membership values determines the fuzzy sets. In certain instances, assigning membership values to fuzzy sets may be challenging. To address the uncertainty around membership values, Turksen suggested the idea of interval-valued fuzzy sets [13]. For accurate representation of an item in a dubious and uncertain state, we need to take membership and non-membership values in particular real-world problems. Atanassaov[2] first developed IFS, which are useful in certain situations. Insufficient data can be handled using IFS, which takes into account both truth and falsehood values.Smarandache was the first to propose the idea of the Neutrosophic set [10]. The neutrosophic set indicates the membership values for truth, uncertainty, and falsehood. Molodstov [7] introduced the concept of a soft set as a brand-new numerical tool for handling problems with ambiguous situations. According to him, a family of universal sets with parameter subsets is a soft set. Soft sets are helpful in many areas, such as artificial intelligence, game theory, and simple decision-making issues [5]. Over the past few years, various researchers have examined the fundamentals of soft set theory. A theoretical analysis of soft sets was by Maji et al. [6] addresses the subset and super set were presented by Ali at el. [1]. Smarandache suggested a fresh approach for dealing with uncertainty. He expanded the Softset($SS$) to Hypersoft($HS$) set and Super Hyper soft set related to the Smarandache power set [12]. Then Jayasudha and Raghavi [4] introduced the concept of Neutrosophic Hypersoft Matrices and their applications in 2024. Hemalatha and Francina Shalini defined the concept of the Pythagorean Neutrosophic Super Hypersoft set [3]. This is closer to our everyday life.

## 2. PRELIMINARIES

Definition 2.1 [11]

$\mathfrak{U}$ , P($\mathfrak{U}$) represents the universe set and power set respectively, Z be parameters. Consider $Å \subset Z$. Then $(F, Å)$ is a soft set over $\mathfrak{U}$ , where $F$: Å ⟶ P($\mathfrak{U}$).

**Definition 2.2 [11]**

$\mathfrak{U}$ −universal set and $\omega$ -set of attributes with respect to $\mathfrak{U}$. Let P($\mathfrak{U}$)-set of Neutrosophic values of $\mathfrak{U}$ and $Å \subseteq \omega$. A pair $(F, Å)$is called a Neutrosophic soft set over $\mathfrak{U}$ and $F$: Å ⟶ P($\mathfrak{U}$).

**Definition 2.3 [9]**

$U$ represent the universe and the power set of $U$ is $\mathfrak{W}(U)$. For t ≥ 1 let $(\mathfrak{S}1, \mathfrak{S}2, \ldots \mathfrak{S}n)_{HS}$ be t-distinct attributes, each of whose associated attributive values is the set $(\mathfrak{s}1, \mathfrak{s}2, \mathfrak{s}3 \ldots \mathfrak{s}t)_{HS}$ with $(\mathfrak{s}y \cap \mathfrak{s}z)_{HS} = \emptyset$ as well as $y \neq z$ and $y, z \in \{1,2, \ldots, t\}$, Then $(\mathfrak{F}, \mathfrak{s}1, \mathfrak{s}2, \mathfrak{s}3 \ldots \mathfrak{s}t)_z$ is a Hypersoft set over $U$.

where $\mathfrak{F}: (\mathfrak{s}1, \mathfrak{s}2, \mathfrak{s}3 \ldots \mathfrak{s}t)_{HS} \longrightarrow \mathfrak{W}(U)$

**Definition 2.4 [8]**

$\mathfrak{U}$ -universal set, P($\mathfrak{U}$)- be a power set of $\mathfrak{U}$. Let $H_1, H_2, H_3 \ldots H_v$ for v≥ 1 be n well -defined attributes, whose corresponding attributive values are respectively the set $h1, h2, h3 \ldots h_v$ with $h_i \cap h_j = \emptyset$ for i≠j and i,j ∈ $\{1,2,3, \ldots, v\}$ and their relation $h_1, h_2, h_3 \ldots = \mathscr{H}$, then the pair (F, $\mathscr{H}$) is said to be Neutrosophic Hypersoft set (PNHSS in short) over $\mathfrak{U}$ where F: $h_1, h_2, h_3 \ldots hv \longrightarrow$ P( $\mathfrak{U}$)

and $(F, h_1, h_2, h_3 \ldots h_v) = \{(\mathcal{H}, <x, T_{F(\mathcal{H})}(x), I_{F(\mathcal{H})}(x), F_{F(\mathcal{H})}(x)>) : x \in \mathfrak{U},\}$ where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $T_{F(\mathscr{H})}(x), I_{F(\mathscr{H})}(x), F_{F(\mathscr{H})}(x) \in [0,1]$ also $0 \leq T_{F(\mathscr{H})}(x), I_{F(\mathscr{H})}(x), F_{F(\mathscr{H})}(x) \leq 3$.

**Definition 2.7 [3]**

$\mathfrak{V}$, P($\mathfrak{V}$) – be the universal set and power set. Consider $H_1, H_2, \ldots H_v$ for v≥ 1 be n-distinct attributes, whose corresponding attributive values are respectively the set $h_1, h_2, \ldots h_v$ with $h_m \cap h_n = \emptyset$ for m≠n and m,n ∈ $\{1,2, \ldots, v\}$. Let P($h_1$) , P($h_2$) , … , P($h_v$) = H be the power sets of the set $h_1, h_2, h_3 \ldots hv$ respectively.

Then the pair ($F$, P($h_1$) × P($h_2$) × P($h_3$) ×, … ,× P($h_v$)) is said to be PNSHSS over $\mathfrak{V}$.

where F: P($h_1$) × P($h_2$) × P($h_3$) ×, … ,× P($h_n$) ⟶ P($\mathfrak{V}$) and

$$F\left(\mathrm{P}(h_1) \times \mathrm{P}(h_2) \times \mathrm{P}(h_3) \times, \ldots, \times \mathrm{P}(h_v)\right)_{PNSHSS} = \left\{ \begin{matrix} \mathcal{H}, < x, T_{F(\mathcal{H})}(x), I_{F(\mathcal{H})}(x), F_{F(\mathcal{H})}(x) >: x \in \mathfrak{V} \\ \mathcal{H} \in \mathrm{P}(h_1) \times \mathrm{P}(h_2) \times \mathrm{P}(h_3) \times, \ldots, \times \mathrm{P}(h_n) \end{matrix} \right\}$$

Where $T_{F(\mathcal{H})}$ and $F_{F(\mathcal{H})}$are the dependent components. $I_{F(\mathcal{H})}$ is independent component.

Also, $0 \leq (TF(\mathscr{H})(x))^2 + (IF(\mathscr{H})(x))^2 + (FF(\mathscr{H})(x))^2 \leq 2$ and $TF(\mathscr{H})(x) + FF(\mathscr{H})(x) \leq 1$.

## 3. PYTHAGOREAN NEUTROSOPHIC SUPER HYPERSOFT MATRICES

$\mathfrak{V} = u^1, u^2, \ldots u^{\gamma}$ be the universal set and P($\mathfrak{V}$) - power set of $\mathfrak{V}$. Consider $\mathfrak{G}1, \mathfrak{G}2, \mathfrak{G}3 \ldots \mathfrak{G}\alpha$ for $\alpha \geq 1$ be $\alpha$ -distinct attributes, whose corresponding attributive values are respectively the set

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$\mathfrak{G}_1^a, \mathfrak{G}_2^b, \mathfrak{G}_3^c \ldots . \mathfrak{G}_\alpha^z$ with $\mathfrak{G}_\mathfrak{r} \cap \mathfrak{G}_\mathfrak{s} = \emptyset$ for $\mathfrak{r} \neq \mathfrak{s}$ and $\mathfrak{r}, \mathfrak{s} \in \{1,2,, \ldots \ldots, \alpha\}$. Let $P(\mathfrak{G}_1^a)$ $P(\mathfrak{G}_2^b), P(\mathfrak{G}_3^c), \ldots \ldots, P(\mathfrak{G}_\alpha^z) = \mathcal{H}$ be the power sets of the set $\mathfrak{G}1\ a$, $\mathfrak{G}2\ b$, $\mathfrak{G}3\ c$ … . . $\mathfrak{G}\alpha\ z$ respectively.

Then the pair $\left(F,\ P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \ldots, \times P(\mathfrak{G}_\alpha^z)\right)$ is said to be PNSHSS over $\mathfrak{B}$.

where $\mathfrak{F}$: $P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z) \longrightarrow P(\mathfrak{B})$ and

$$\mathfrak{F}\left(P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z)\right) = \left\{ \begin{matrix} \mathcal{H}, < x, T_{\mathfrak{F}(\mathcal{H})}(x), I_{\mathfrak{F}(\mathcal{H})}(x), F_{\mathfrak{F}(\mathcal{H})}(x) >: x \in \mathfrak{B}, \\ \mathcal{H} \in P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z) \end{matrix} \right\}$$

Where $T_{\mathfrak{F}(\mathcal{H})}$ and $F_{\mathfrak{F}(\mathcal{H})}$ are the dependent components. $I_{\mathfrak{F}(\mathcal{H})}$ is independent component. Also,

$$0 \leq \left(T_{\mathfrak{F}(\mathcal{H})}(x)\right)^2 + \left(I_{\mathfrak{F}(\mathcal{H})}(x)\right)^2 + \left(F_{\mathfrak{F}(\mathcal{H})}(x)\right)^2 \leq 2 \text{ and } T_{\mathfrak{F}(\mathcal{H})}(x) + F_{\mathfrak{F}(\mathcal{H})}(x) \leq 1.$$

Let $H_i = P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z)$

$$x_{\mathfrak{H}_i} = P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z)$$

It is defined as $x_{\mathfrak{H}_i} = \{x, T_{\mathfrak{F}(H)}(x), I_{\mathfrak{F}(H)}(x), F_{\mathfrak{F}(H)}(x)\}\ x \in \mathfrak{B}, i \in P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z)$

and can be representation of $\mathfrak{H}i$ as given in the table

| U | $P(\mathfrak{G}_1^a)$ | $P(\mathfrak{G}_2^b)$ | ⋯ | $P(\mathfrak{G}_\alpha^z)$ |
|---|---|---|---|---|
| $u^1$ | $x_{\mathfrak{H}_i}(u^1, P(\mathfrak{G}_1^a))$ | $x_{\mathfrak{H}_i}(u^1, P(\mathfrak{G}_2^b))$ | ⋯ | $x_{\mathfrak{H}_i}(u^1, P(\mathfrak{G}_\alpha^z))$ |
| $u^2$ | $x_{\mathfrak{H}_i}(u^2, P(\mathfrak{G}_1^a))$ | $x_{\mathfrak{H}_i}(u^2, P(\mathfrak{G}_2^b))$ | ⋯ | $x_{\mathfrak{H}_i}(u^2, P(\mathfrak{G}_\alpha^z))$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $u^\delta$ | $x_{\mathfrak{H}_i}(u^\delta, P(\mathfrak{G}_1^a))$ | $x_{\mathfrak{H}_i}(u^\delta, P(\mathfrak{G}_2^b))$ | ⋯ | $x_{\mathfrak{H}_i}(u^\delta, P(\mathfrak{G}_\alpha^z))$ |

If $M_{st} = x_{\mathfrak{H}_i}\left(u^\ell, P(\mathfrak{G}_m^n)\right)$

*Where* $\ell = 1,2, \cdots \delta,\ n = a, b, \cdots z,\ m = 1,2, \cdots \alpha$

Then the matrix is defined as,

$$[M_{st}]_{\alpha \times n} = \begin{pmatrix} M_{11} & M_{12} & \ldots & M \\ M_{21} & M_{22} & \ldots & M_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ M_{\alpha 1} & M & \ldots & M_{\alpha n} \end{pmatrix}$$

Where $M_{st} = \left(T_{P(\mathfrak{G}_m^n)}(u_i), I_{P(\mathfrak{G}_m^n)}(u_i), F_{P(\mathfrak{G}_m^n)}(u_i), u_i \in U, P(\mathfrak{G}_m^n) \in P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z)\right) = (T_{stn}^M, I_{stn}^M, F_{stn}^M)$

By using the Pythagorean Neutrosophic Super Hypersoft matrix (PNSHM), we may therefore express any Pythagorean Neutrosophic Super Hypersoft Set.

**Example.1**

Let $\gamma$ be the collection of mobile phones within the showroom:

$\gamma = \{Lava, Tecno, Poco, iTel\}$

The decision maker presents their conclusion on the selection process of the alternatives, including:

$\gamma = \{Lava, Tecno, Poco, iTel\}$

$$\mathfrak{G}_1 = \{Storage\} = \mathfrak{G}_1^a = \{64GB, 128GB, 256GB\}$$
$$\mathfrak{G}_2 = \{Camera\ Resolution\} = \mathfrak{G}_2^b = \{20Mp, 50Mp\}$$
$$\mathfrak{G}_3 = \{Battery\} = \mathfrak{G}_3^c = \{4200mAh, 4800mAh\}$$
$$\mathfrak{G}_4 = \{RAM\} = \mathfrak{G}_4^d = \{6GB, 8GB, 12GB\}$$
$$P(\mathfrak{G}_1^a) = \left\{ \begin{matrix} \{64GB\}, \{128GB\}, \{256GB\}, \{64GB, 128GB\}\{64GB, 256GB\}, \\ \{128GB, 256GB\}, \{64GB, 128GB, 256GB\}, \emptyset \end{matrix} \right\}$$
$$P(\mathfrak{G}_2^b) = \{\{20Mp\}, \{50Mp\}, \{20Mp, 50Mp\}, \emptyset\}$$
$$P(\mathfrak{G}_3^c) = \{\{4200\ mAh\}, \{4800\ mAh\}, \{4200\ mAh, 4800\ mAh\}, \emptyset\}$$
$$P(\mathfrak{G}_4^d) = \{\{6GB\}, \{8GB\}, \{12GB\}, \{6GB, 8\ GB\}, \{6GB, 12GB\}, \{8GB, 12\ GB\}, \{6GB, 8GB, 12\ GB\}, \emptyset\}$$

Let $S: P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z) \rightarrow P(\gamma)$

Pythagorean Neutrosophic Super Hypersoft set is characterized as:

$$S: P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \ldots \ldots, \times P(\mathfrak{G}_\alpha^z) \rightarrow P(\gamma)$$

| | Lava | Tecno | Poco | iTel |
|---|---|---|---|---|
| 6GB | (0.2, 0.7, 0.1) | (0.3, 0.1, 0.5) | (0.5, 0.3, 0.5) | (0.6, 0.2, 0.1) |
| 128GB | (0.3, 0.5, 0.2) | (0.7, 0.1, 0.2) | (0.6, 0.2, 0.3) | (0.8, 0.5, 0.1) |
| 256 GB | (0.4, 0.3, 0.6) | (0.6, 0.5, 0.4) | (0.7, 0.1, 0.2) | (0.5, 0.2, 0.3) |
| {64 GB, 128 GB} | (0.3, 0.5, 0.6) | (0.6, 0.5, 0.4) | (0.6, 0.2, 0.3) | (0.5, 0.2, 0.3) |
| {64 GB, 256 GB} | (0.3, 0.5, 0.2) | (0.6, 0.1, 0.4) | (0.6, 0.2, 0.3) | (0.6, 0.5, 0.1) |
| {128 GB, 256 GB} | (0.2, 0.7, 0.2) | (0.3, 0.1, 0.5) | (0.5, 0.3, 0.5) | (0.6, 0.2, 0.1) |
| {64 GB, 128 GB, 256 GB} | (0.4, 0.3, 0.1) | (0.7, 0.1, 0.2) | (0.7, 0.1, 0.2) | (0.8, 0.2, 0.1) |
| ∅ | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) |
| 20 Mp | (0.3, 0.6, 0.6) | (0.2, 0.4, 0.6) | (0.4, 0.5, 0.4) | (0.5, 0.2, 0.2) |
| 50 Mp | (0.1, 0.7, 0.4) | (0.6, 0.5, 0.2) | (0.3, 0.1, 0.2) | (0.8, 0.3, 0.1) |
| {20 Mp, 50 Mp} | (0.3, 0.6, 0.4) | (0.6, 0.4, 0.2) | (0.4, 0.1, 0.2) | (0.8, 0.2, 0.1) |
| ∅ | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) |

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| | | | | |
|---|---|---|---|---|
| 4200 mAh | (0.6, 0.1, 0.2) | (0.3, 0.2, 0.6) | (0.1, 0.7, 0.5) | (0.3, 0.4, 0.5) |
| 4800 mAh | (0.7, 0.2, 0.3) | (0.5, 0.3, 0.4) | (0.6, 0.1, 0.4) | (0.5, 0.4, 0.3) |
| {4200 mAh, 4800 mAh} | (0.7, 0.1, 0.2) | (0.5, 0.2, 0.4) | (0.6, 0.1, 0.4) | (0.5, 0.4, 0.3) |
| ∅ | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) |
| 6GB | (0.6, 0.5, 0.3) | (0.5, 0.5, 0.5) | (0.9, 0.1, 0.1) | (0.6, 0.7, 0.1) |
| 8 GB | (0.8, 0.4, 0.1) | (0.5, 0.2, 0.5) | (0.8, 0.1, 0.2) | (0.1, 0.6, 0.7) |
| 12 GB | (0.9, 0.1, 0.1) | (0.8, 0.5, 0.1) | (0.6, 0.5, 0.3) | (0.3, 0.5, 0.2) |
| {6 GB, 8 GB} | (0.8, 0.4, 0.1) | (0.5, 0.5, 0.5) | (0.6, 0.1, 0.3) | (0.3, 0.6, 0.2) |
| {6 GB, 12 GB} | (0.8, 0.4, 0.1) | (0.5, 0.5, 0.5) | (0.8, 0.1, 0.2) | (0.1, 0.6, 0.7) |
| {8 GB, 12 GB} | (0.6, 0.5, 0.3) | (0.5, 0.5, 0.5) | (0.8, 0.1, 0.2) | (0.3, 0.7, 0.2) |
| {6GB, 8GB, 12 GB} | (0.9, 0.1, 0.1) | (0.8, 0.2, 0.1) | (0.9, 0.1, 0.1) | (0.6, 0.5, 0.1) |
| ∅ | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) |

Let us assume $\mathfrak{F}: P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \dots\dots, \times P(\mathfrak{G}_\alpha^z) =$

$$S(256\,GB, \{20\,Mp, 150\,Mp\}, \{4200\,mAh, 4800\,mAh\}, 8GB) = \{Techno, Poco\}$$

Therefore, the connection's PNSH that is expected is:

$$S: P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \dots\dots, \times P(\mathfrak{G}_\alpha^z)$$
$$= S(256\,GB, \{20Mp, 50Mp\}, \{4200\,mAh, 4800\,mAh\}, 8GB)$$
$$= \left\{ \begin{matrix} (Tecno, (0.6, 0.5, 0.4), (0.6, 0.4, 0.2), (0.5, 0.2, 0.4), (0.5, 0.2, 0.5)), \\ (Poco, (0.6, 0.1, 0.4), (0.4, 0.1, 0.2), (0.6, 0.1, 0.4), (0.8, 0.1, 0.2)) \end{matrix} \right\}$$

Additionally, it may be shown as follows in matrix form:

$$[S]_{2\times4} = \begin{pmatrix} (0.6, 0.5, 0.4) & (0.6, 0.4, 0.2) & (0.5, 0.2, 0.4) & (0.5, 0.2, 0.5) \\ (0.7, 0.1, 0.2) & (0.4, 0.1, 0.2) & (0.6, 0.1, 0.4) & (0.8, 0.1, 0.2) \end{pmatrix}$$

**Definition**

Let $S = [s_{st}]$ and $\mathbb{P} = [p_{st}]$ be two Pythagorean Neutrosophic Super Hypersoft matrix Set with order $\partial \times \mathcal{E}$, where $S_{st} = (T_{stn}^s, I_{stn}^s, F_{stn}^s)$ and $P_{st} = (T_{stn}^p, I_{stn}^p, F_{stn}^p)$. S is said to Pythagorean Neutrosophic Super Hypersoft sub matrix of P if $T_{stn}^s \le T_{stn}^p, I_{stn}^s \le I_{stn}^p, F_{stn}^s \ge F_{stn}^p$.

**Example 2**

Consider PNSHS Matrix $[S]_{2\times4}$ of example 1.

$$[S]_{2\times4} = \begin{pmatrix} (0.6, 0.5, 0.4) & (0.6, 0.4, 0.2) & (0.5, 0.2, 0.4) & (0.5, 0.2, 0.5) \\ (0.7, 0.1, 0.2) & (0.4, 0.1, 0.2) & (0.6, 0.1, 0.4) & (0.8, 0.1, 0.2) \end{pmatrix}$$

Consider another PNSHS matrix [ℙ] related with the Pythagorean Neutrosophic Super Hypersoft Set.

$$\mathbb{P}: P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times P(\mathfrak{G}_3^c) \times, \dots\dots, \times P(\mathfrak{G}_\alpha^z) \to P(\gamma)$$

The universe and attributes mentioned in Example 1 will remain the same

$$P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \dots\dots, \times P(\mathfrak{G}_\alpha^z) = \mathbb{P}(256\,GB, \{20\,Mp, 50\,Mp\}, \{4200\,mAh, 4800\,mAh\}, 8GB)$$
$$= \left\{ \begin{matrix} (Tecno, (0.7, 0.6, 0.3), (0.8, 0.5, 0.1), (0.7, 0.3, 0.3), (0.8, 0.5, 0.1)), \\ (Pocco, (0.8, 0.5, 0.1), (0.6, 0.3, 0.1), (0.7, 0.5, 0.1), (0.9, 0.3, 0.2)) \end{matrix} \right\}$$

Hence the PNSHS matrix $[P]$ is written as,

$$[\mathbb{P}]_{2x4} = \begin{pmatrix} (0.7,0.6,0.3) & (0.8,0.5,0.1) & (0.7,0.3,0.3) & (0.8,0.5,0,1) \\ (0.8,0.5,0.1) & (0.6,0.3,0.2) & (0.7,0.5,0.1) & (0.9,0.3,0.2) \end{pmatrix}$$

The Pythagorean Neutrosophic Super Hypersoft Sub matrix condition is thus satisfied by the membership values of "Tecno" and "Poco" for 256GB in both sets, which are (0.7,0.6,0.3) and (0.8,0.5,0.1), respectively. This demonstrates that (0.7,0.6,0.3)⊆ (0.8,0.5,0.1), and the remaining $[S]$ $amd$ $PNSHM$[ℙ] qualities were similarly affected.

**Definition**

Take $[S] = [s_{st}]$ and $[\mathbb{P}] = [p_{st}]$ be two Pythagorean Neutrosophic Super Hyper soft Matrix with order $\partial \times \mathcal{E}$, where $s_{st} = (T_{stn}^s, I_{stn}^s, F_{stn}^s)$ and $p_{st} = (T_{stn}^p, I_{stn}^p, F_{stn}^p)$. Then S is said to be the Equal PNSHM of ℙ if,

$$T_{stn}^s = T_{stn}^p, \quad I_{stn}^s = I_{stn}^p, \quad F_{stn}^s = F_{stn}^p$$

**Example 3**

Consider PNSHS Matrix $[S]_{2\times4}$ of example 1.

$$[S]_{2\times4} = \begin{pmatrix} (0.6, 0.5, 0.4) & (0.6, 0.4, 0.2) & (0.5, 0.2, 0.4) & (0.5, 0.2, 0.5) \\ (0.7, 0.1, 0.2) & (0.4, 0.1, 0.2) & (0.6, 0.1, 0.4) & (0.8, 0.1, 0.2) \end{pmatrix}$$

Let's look at another PNSHM [ℙ] that is associated with the Pythagorean Neutrosophic Super Hypersoft set $\mathbb{P}: P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \dots\dots, \times P(\mathfrak{G}_\alpha^z) \to P(\gamma)$ over the same universe and characteristics as seen in Example 1.

$$\left(P(\mathfrak{G}_1^a) \times P(\mathfrak{G}_2^b) \times, \dots\dots, \times P(\mathfrak{G}_\alpha^z)\right)$$
$$= \mathbb{P}(256\,GB, \{20\,Mp, 50\,Mp\}, \{4200\,mAh, 4800\,mAh\}, 8GB)$$
$$= \left\{ \begin{matrix} (Tecno, (0.6, 0.5, 0.4), (0.6, 0.4, 0.2), (0.5, 0.2, 0.4), (0.5, 0.2, 0.5)), \\ (Pocco, (0.7, 0.1, 0.2), (0.4, 0.1, 0.2), (0.6, 0.1, 0.4), (0.8, 0.1, 0.2)) \end{matrix} \right\}$$

Then PNSHM [ℙ] is written as,

$$[\mathbb{P}]_{2\times4} = \begin{pmatrix} (0.6, 0.5, 0.4) & (0.6, 0.4, 0.2) & (0.5, 0.2, 0.4) & (0.5, 0.2, 0.5) \\ (0.7, 0.1, 0.2) & (0.4, 0.1, 0.2) & (0.6, 0.1, 0.4) & (0.8, 0.1, 0.2) \end{pmatrix}$$

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Consequently, we can see that the membership values of "Tecno" and "Pocco" for 256GB in both matrices meet the Equal PNSHM criteria, which are 0.6=0.6, 0.5=0.5, 0.4=0.4, and 0.7=0.7, 0.1=0.1, 0.2=0.2. As de monstrated by this, (0.6,0.5,0.4) = (0.6,0.5,0.4) and (0.7,0.1,0.2) = (0.7,0.1,0.2), and the same was true for the remaining PNSHM characteristics $[S]$ *and* $[\mathbb{P}]$.

**Definition**

Let $S = [\mathcal{s}t]$ be Pythagorean Neutrosophic Super Hypersoft matrix Set with order $\partial \times \mathcal{E}$, where $\mathcal{s}_{\mathcal{s}t} = (T^{s}_{\mathcal{s}tn}, I^{s}_{\mathcal{s}tn}, F^{s}_{\mathcal{s}tn})$. Then S-Null Pythagorean Neutrosophic Super Hypersoft matrix if

$$T^{s}_{\mathcal{s}tn} = 0, I^{s}_{\mathcal{s}tn} = 1, F^{s}_{\mathcal{s}tn} = 1.$$

**Example 4**

A Null-PNSHM S is then provided by taking the same universe and characteristics as in Example 1.

$$[S]_{2\times4} = \begin{pmatrix} (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \\ (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \end{pmatrix}$$

**Definition**

Let $S = [\mathcal{s}_{\mathcal{s}t}]$ be Pythagorean Neutrosophic Super Hypersoft matrix Set with order $\partial \times \mathcal{E}$, where $\mathcal{s}_{\mathcal{s}t} = (T^{s}_{ijk}, I^{s}_{ijk}, F^{s}_{ijk})$. Consequently, if matrix S is a Universal Pythagorean Neutrosophic Super Hypersoft matrix, then

$$T^{s}_{ijk} = 1, I^{s}_{ijk}0, F^{s}_{ijk} = 0.$$

**Example 5**

A Universal-PNSHM S is then provided by taking the same universe and characteristics as in Example 1.

$$[S]_{2\times4} = \begin{pmatrix} (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \\ (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \end{pmatrix}$$

**Definition**

Let $S = [\mathcal{s}_{\mathcal{s}t}]$ be Pythagorean Neutrosophic Super Hypersoft matrix Set with order $\partial \times \mathcal{E}$, where

$\mathcal{s}_{\mathcal{s}t} = (T^{s}_{\mathcal{s}tn}, I^{s}_{\mathcal{s}tn}, F^{s}_{\mathcal{s}tn})$. Then $S = [\mathcal{s}_{\mathcal{s}t}]^c$ the matrix S is said to be Complement Pythagorean Neutrosophic Super Hypersoft matrix of $S = [\mathcal{s}_{\mathcal{s}t}]$ if $(T^{s}_{\mathcal{s}tn}, I^{s}_{\mathcal{s}tn}, F^{s}_{\mathcal{s}tn})^c = (F^{s}_{\mathcal{s}tn}, I^{s}_{\mathcal{s}tn}, T^{s}_{\mathcal{s}tn})$

**Example**

6 Consider PNSHS Matrix $[S]2\times4$ of example 1.

$$[S]_{2\times4} = \begin{pmatrix} (0.6,0.5,0.4) & (0.6,0.4,0.2) & (0.5,0.2,0.4) & (0.5,0.2,0.5) \\ (0.7,0.1,0.2) & (0.4,0.1,0.2) & (0.6,0.1,0.4) & (0.8,0.1,0.2) \end{pmatrix}$$

Then

$$[S]^{c}{}_{2\times4} = \begin{pmatrix} (0.4,0.5,0.6) & (0.2,0.4,0.6) & (0.4,0.2,0.5) & (0.5,0.2,0.5) \\ (0.2,0.1,0.7) & (0.2,0.1,0.4) & (0.4,0.1,0.6) & (0.2,0.1,0.8) \end{pmatrix}$$

## 4. PROPERTIES OF PNSHM

**Proposition 4.1**

Let $S_{\mathfrak{y}} = [\mathcal{s}_{\mathcal{s}t}]$ , $\mathbb{P}_{\mathfrak{y}} = [p_{\mathcal{s}t}]$ and $O_{\mathfrak{y}} = [o_{\mathcal{s}t}]$ where $\mathcal{s}_{\mathcal{s}t} = (T^{s}_{\mathcal{s}tn}, I^{s}_{\mathcal{s}tn}, F^{s}_{\mathcal{s}tn}), p_{\mathcal{s}t} = (T^{p}_{\mathcal{s}tn}, I^{p}_{\mathcal{s}tn}, F^{p}_{\mathcal{s}tn})$ *and* $o_{\mathcal{s}t} = (T^{o}_{\mathcal{s}tn}, I^{o}_{\mathcal{s}tn}, F^{o}_{\mathcal{s}tn}) \in$ PNSHMs with order $\partial \times \mathcal{E}$. Then,

i. $S_{\mathfrak{y}} \otimes (\mathbb{P}_{\mathfrak{y}} \cup O_{\mathfrak{y}}) = (S_{\mathfrak{y}} \otimes \mathbb{P}_{\mathfrak{y}}) \cup (S_{\mathfrak{y}} \otimes O_{\mathfrak{y}}),\ (S_{\mathfrak{y}} \cup \mathbb{P}_{\mathfrak{y}}) \otimes O_{\mathfrak{y}} = (S_{\mathfrak{y}} \otimes \mathbb{P}_{\mathfrak{y}}) \cup (S_{\mathfrak{y}} \otimes O_{\mathfrak{y}})$
ii. $S_{\mathfrak{y}} \otimes (\mathbb{P}_{\mathfrak{y}} \cap O_{\mathfrak{y}}) = (S_{\mathfrak{y}} \otimes \mathbb{P}_{\mathfrak{y}}) \cap (S_{\mathfrak{y}} \otimes O_{\mathfrak{y}}),\ (S_{\mathfrak{y}} \cap \mathbb{P}_{\mathfrak{y}}) \otimes O_{\mathfrak{y}} = (S_{\mathfrak{y}} \otimes \mathbb{P}_{\mathfrak{y}}) \cap (S_{\mathfrak{y}} \otimes O_{\mathfrak{y}})$
iii. $S_{\mathfrak{y}} \boxtimes (\mathbb{P}_{\mathfrak{y}} \cup O_{\mathfrak{y}}) \neq (S_{\mathfrak{y}} \boxtimes \mathbb{P}_{\mathfrak{y}}) \cup (S_{\mathfrak{y}} \boxtimes O_{\mathfrak{y}}),\ (S_{\mathfrak{y}} \cup \mathbb{P}_{\mathfrak{y}}) \boxtimes O_{\mathfrak{y}} \neq (S_{\mathfrak{y}} \boxtimes \mathbb{P}_{\mathfrak{y}}) \cup (S_{\mathfrak{y}} \boxtimes O_{\mathfrak{y}})$
iv. $S_{\mathfrak{y}} \boxtimes (\mathbb{P}_{\mathfrak{y}} \cap O_{\mathfrak{y}}) \neq (S_{\mathfrak{y}} \boxtimes \mathbb{P}_{\mathfrak{y}}) \cap (S_{\mathfrak{y}} \boxtimes O_{\mathfrak{y}}),\ (S_{\mathfrak{y}} \cap \mathbb{P}_{\mathfrak{y}}) \boxtimes O_{\mathfrak{y}} \neq (S_{\mathfrak{y}} \boxtimes \mathbb{P}_{\mathfrak{y}}) \cap (S_{\mathfrak{y}} \boxtimes O_{\mathfrak{y}})$
v. $S_{\mathfrak{y}} \blacksquare (\mathbb{P}_{\mathfrak{y}} \cup O_{\mathfrak{y}}) \neq (S_{\mathfrak{y}} \blacksquare \mathbb{P}_{\mathfrak{y}}) \cup (S_{\mathfrak{y}} \blacksquare O_{\mathfrak{y}}),\ (S_{\mathfrak{y}} \cup \mathbb{P}_{\mathfrak{y}}) \blacksquare O_{\mathfrak{y}} \neq (S_{\mathfrak{y}} \blacksquare \mathbb{P}_{\mathfrak{y}}) \cup (S_{\mathfrak{y}} \blacksquare O_{\mathfrak{y}})$
vi. $S_{\mathfrak{y}} \blacksquare (\mathbb{P}_{\mathfrak{y}} \cap O_{\mathfrak{y}}) \neq (S_{\mathfrak{y}} \blacksquare \mathbb{P}_{\mathfrak{y}}) \cap (S_{\mathfrak{y}} \blacksquare O_{\mathfrak{y}}),\ (S_{\mathfrak{y}} \cap \mathbb{P}_{\mathfrak{y}}) \blacksquare O_{\mathfrak{y}} \neq (S_{\mathfrak{y}} \blacksquare \mathbb{P}_{\mathfrak{y}}) \cap (S_{\mathfrak{y}} \blacksquare O_{\mathfrak{y}})$

**Proof**

i. $S_{\mathfrak{y}} \otimes (\mathbb{P}_{\mathfrak{y}} \cup O_{\mathfrak{y}}),\ (S_{\mathfrak{y}} \otimes \mathbb{P}_{\mathfrak{y}}) \cup (S_{\mathfrak{y}} \otimes O_{\mathfrak{y}}) \in PNSHM\ with\ order\ \partial \times \mathcal{E}$. Then

$$S_{\mathfrak{y}} \otimes (\mathbb{P}_{\mathfrak{y}} \cup O_{\mathfrak{y}}) = [(T^{s}_{\mathcal{s}tn}, I^{s}_{\mathcal{s}tn}, F^{s}_{\mathcal{s}tn})] \otimes \left[\left(\max(T^{p}_{\mathcal{s}tn}, T^{o}_{\mathcal{s}tn}), \frac{(I^{p}_{\mathcal{s}tn}, I^{o}_{\mathcal{s}tn})}{2}, min(F^{p}_{\mathcal{s}tn}, F^{o}_{\mathcal{s}tn})\right)\right]$$

$$= \left[\left(\frac{T^{s}_{\mathcal{s}tn} + max(T^{p}_{\mathcal{s}tn}, T^{o}_{\mathcal{s}tn})}{2}, \frac{I^{s}_{\mathcal{s}tn} + \frac{(I^{p}_{\mathcal{s}tn}, I^{o}_{\mathcal{s}tn})}{2}}{2}, \frac{F^{s}_{\mathcal{s}tn} + min(F^{p}_{\mathcal{s}tn}, F^{o}_{\mathcal{s}tn})}{2}\right)\right]$$

$$=\left[\left(max\left[\left(\frac{(T^s_{stn},I^p_{stn})}{2}\right),\left(\frac{(I^s_{stn},I^o_{stn})}{2}\right)\right],\frac{\frac{(I^s_{stn},I^p_{ijk})}{2}+\frac{(I^s_{stn},I^o_{stn})}{2}}{2},min\left[\left(\frac{(F^s_{stn},F^p_{stn})}{2}\right),\left(\frac{(F^s_{stn},F^o_{stn})}{2}\right)\right]\right)\right]$$

$$=\left[\left(\frac{(T^s_{stn},T^p_{stn})}{2},\frac{(I^s_{stn},I^p_{stn})}{2},\frac{(F^s_{stn},F^p_{stn})}{2}\right)\right]\cup\left[\left(\frac{(T^s_{stn},T^o_{stn})}{2},\frac{(I^s_{stn},I^o_{stn})}{2},\frac{(F^s_{stn},F^o_{stn})}{2}\right)\right]$$

$$S_{\mathfrak{y}}\otimes(\mathbb{P}_{\mathfrak{y}}\cup O_{\mathfrak{y}})=(S_{\mathfrak{y}}\otimes\mathbb{P}_{\mathfrak{y}})\cup(S_{\mathfrak{y}}\otimes O_{\mathfrak{y}})$$

ii. $S_{\mathfrak{y}}\otimes(\mathbb{P}_{\mathfrak{y}}\cap O_{\mathfrak{y}}),(S_{\mathfrak{y}}\otimes\mathbb{P}_{\mathfrak{y}})\cap(S_{\mathfrak{y}}\otimes O_{\mathfrak{y}})\in PNSHM\ with\ order\ \partial\times\mathcal{E}$. Then

$$S_{\mathfrak{y}}\otimes(\mathbb{P}_{\mathfrak{y}}\cap O_{\mathfrak{y}})=[(T^s_{stn},I^s_{stn},F^s_{stn})]\otimes\left[\left(\min(T^p_{stn},T^o_{stn}),\frac{(I^p_{stn},I^o_{stn})}{2},max(F^p_{stn},F^o_{stn})\right)\right]$$

$$=\left[\left(\frac{T^s_{stn}+min(T^p_{stn},T^o_{stn})}{2},\frac{I^s_{stn}+\frac{(I^p_{stn},I^o_{stn})}{2}}{2},\frac{F^s_{stn}+max(F^p_{stn},F^o_{stn})}{2}\right)\right]$$

$$=\left[\left(min\left[\left(\frac{(T^s_{stn},I^p_{stn})}{2}\right),\left(\frac{(T^s_{stn},T^o_{stn})}{2}\right)\right],\frac{\frac{(I^s_{stn},I^p_{stn})}{2}+\frac{(I^s_{stn},I^o_{stn})}{2}}{2},max\left[\left(\frac{(F^s_{stn},F^p_{stn})}{2}\right),\left(\frac{(F^s_{stn},F^o_{stn})}{2}\right)\right]\right)\right]$$

$$=\left[\left(\frac{(T^s_{stn},I^p_{stn})}{2},\frac{(I^s_{stn},I^p_{stn})}{2},\frac{(F^s_{stn},F^p_{stn})}{2}\right)\right]\cap\left[\left(\frac{(T^s_{stn},T^o_{stn})}{2},\frac{(I^s_{stn},I^o_{stn})}{2},\frac{(F^s_{stn},F^o_{stn})}{2}\right)\right]$$

$$S_{\mathfrak{y}}\otimes(\mathbb{P}_{\mathfrak{y}}\cap O_{\mathfrak{y}})=(S_{\mathfrak{y}}\otimes\mathbb{P}_{\mathfrak{y}})\cap(S_{\mathfrak{y}}\otimes O_{\mathfrak{y}})$$

iii. Here $(S_{\mathfrak{y}}\cup\mathbb{P}_{\mathfrak{y}})\boxtimes O_{\mathfrak{y}},(S_{\mathfrak{y}}\boxtimes\mathbb{P}_{\mathfrak{y}})\cup(S_{\mathfrak{y}}\boxtimes O_{\mathfrak{y}})\in PNSHM\ with\ order\ \partial\times\mathcal{E}$. Then $(S_{\mathfrak{y}}\cup\mathbb{P}_{\mathfrak{y}})\boxtimes O_{\mathfrak{y}}$

$$=\left[\left(\max(T^s_{stn},T^p_{stn}),\frac{(I^s_{stn},I^p_{stn})}{2},min(F^s_{stn},F^p_{stn})\right)\right]\boxtimes[(T^o_{stn},I^o_{stn},F^o_{stn})]$$

$$=\left[\left(\sqrt{\max(T^s_{stn},T^p_{stn}).T^o_{ijk}},\sqrt{\frac{(I^s_{stn},I^p_{stn})}{2}.I^o_{stn}},\sqrt{min(F^s_{stn},F^p_{stn}).F^o_{stn}}\right)\right]$$

$$=\left[\left(\begin{array}{c}max\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right),\left(\sqrt{\frac{(I^s_{stn},I^o_{stn})}{2}}+\sqrt{\frac{(I^p_{stn},I^o_{stn})}{2}}\right),\\ min\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right)\end{array}\right)\right]$$

$(S_{\mathfrak{y}}\boxtimes\mathbb{P}_{\mathfrak{y}})\cup(S_{\mathfrak{y}}\boxtimes O_{\mathfrak{y}})$

$$=\left[\left(max\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right),\left(\frac{\sqrt{(I^s_{stn},I^o_{stn})}+\sqrt{(I^p_{stn},I^o_{stn})}}{2}\right),min\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right)\right)\right]$$

Therefore, we have $(S_{\mathfrak{y}}\cup\mathbb{P}_{\mathfrak{y}})\boxtimes O_{\mathfrak{y}}\neq(S_{\mathfrak{y}}\boxtimes\mathbb{P}_{\mathfrak{y}})\cup(S_{\mathfrak{y}}\boxtimes O_{\mathfrak{y}})$.

iv. Here $(S_{\mathfrak{y}}\cap\mathbb{P}_{\mathfrak{y}})\boxtimes O_{\mathfrak{y}},(S_{\mathfrak{y}}\boxtimes\mathbb{P}_{\mathfrak{y}})\cap(S_{\mathfrak{y}}\boxtimes O_{\mathfrak{y}})\in PNSHM\ with\ order\ \partial\times\mathcal{E}$. Then

$$(S_{\mathfrak{y}}\cap\mathbb{P}_{\mathfrak{y}})\boxtimes O_{\mathfrak{y}}=\left[\left(\min(T^s_{stn},T^p_{stn}),\frac{(I^s_{stn},I^p_{stn})}{2},max(F^s_{stn},F^p_{stn})\right)\right]\boxtimes[(T^o_{stn},I^o_{stn},F^o_{stn})]$$

$$=\left[\left(\sqrt{\min(T^s_{stn},T^p_{stn}).T^o_{stn}},\sqrt{\frac{(I^s_{stn},I^p_{stn})}{2}.I^o_{stn}},\sqrt{max(F^s_{stn},F^p_{stn}).F^o_{stn}}\right)\right]$$

$$=\left[\left(min\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right),\left(\sqrt{\frac{(I^s_{stn},I^o_{stn})}{2}}+\sqrt{\frac{(I^p_{stn},I^o_{stn})}{2}}\right),max\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right)\right)\right]$$

$(S\boxtimes\mathbb{P})\cap(S\boxtimes O_{\mathfrak{y}})$

$$=\left[\left(min\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right),\left(\frac{\sqrt{(I^s_{stn},I^o_{stn})}+\sqrt{(I^p_{stn},I^o_{stn})}}{2}\right),max\left(\sqrt{(T^s_{stn},T^o_{stn})},\sqrt{(T^p_{stn},T^o_{stn})}\right)\right)\right]$$

Therefore, we have $(S_{\mathfrak{y}}\cap\mathbb{P}_{\mathfrak{y}})\boxtimes O_{\mathfrak{y}}\neq(S_{\mathfrak{y}}\boxtimes\mathbb{P}_{\mathfrak{y}})\cap(S_{\mathfrak{y}}\boxtimes O_{\mathfrak{y}})$.

v. Here $(S_{\mathfrak{y}}\cup\mathbb{P}_{\mathfrak{y}})\blacksquare O_{\mathfrak{y}},\ (S_{\mathfrak{y}}\blacksquare\mathbb{P}_{\mathfrak{y}})\cup(S_{\mathfrak{y}}\blacksquare O_{\mathfrak{y}})\in PNSHM\ with\ order\ \partial\times\mathcal{E}$. Then

$$(S_{\mathfrak{y}}\cup\mathbb{P}_{\mathfrak{y}})\blacksquare O_{\mathfrak{y}}=\left[\left(\max(T^s_{stn},T^p_{stn}),\frac{(I^s_{stn},I^p_{stn})}{2},min(F^s_{stn},F^p_{stn})\right)\right]\blacksquare[(T^o_{stn},I^o_{stn},F^o_{stn})]$$

$$=\left[\left(\frac{\max(T^s_{stn},T^p_{stn}).2T^o_{ijk}}{\max(T^s_{stn},T^p_{stn})+T^o_{stn}},\frac{\frac{(I^s_{stn},I^p_{stn})}{2}.2I^o_{ijk}}{\frac{(I^s_{stn},I^p_{stn})}{2}+I^o_{stn}},\frac{min(F^s_{stn},F^p_{stn}).2F^o_{stn}}{min(F^s_{stn},F^p_{stn})+F^o_{stn}}\right)\right]$$

$(S_{\mathfrak{y}}\cup\mathbb{P}_{\mathfrak{y}})\blacksquare O_{\mathfrak{y}}$

$$=\left[\left(max\left(\frac{2T^s_{stn}.T^o_{stn}}{T^s_{stn}+T^o_{stn}},\frac{2T^p_{stn}.T^o_{stn}}{T^p_{stn}+T^o_{stn}}\right),\left(\frac{2I^s_{stn}.I^o_{stn}}{I^s_{stn}+I^o_{stn}},\frac{2I^p_{stn}.T^o_{stn}}{I^p_{stn}+I^o_{stn}}\right),min\left(\frac{2F^s_{stn}.F^o_{stn}}{F^s_{stn}+F^o_{stn}},\frac{2F^p_{stn}.F^o_{stn}}{F^p_{stn}+F^o_{stn}}\right)\right)\right]$$

$(S_{\mathfrak{y}}\blacksquare\mathbb{P}_{\mathfrak{y}})\cup(S_{\mathfrak{y}}\blacksquare O_{\mathfrak{y}})$

$$=\left[\left(max\left(\frac{2T^s_{stn}.T^o_{stn}}{T^s_{stn}+T^o_{stn}},\frac{2T^p_{stn}.T^o_{stn}}{T^p_{stn}+T^o_{stn}}\right),\frac{\left(\frac{2I^s_{stn}.I^o_{stn}}{I^s_{stn}+I^o_{stn}},\frac{2I^p_{stn}.T^o_{stn}}{I^p_{stn}+I^o_{stn}}\right)}{2},min\left(\frac{2F^s_{stn}.F^o_{stn}}{F^s_{stn}+F^o_{stn}},\frac{2F^p_{stn}.F^o_{stn}}{F^p_{stn}+F^o_{stn}}\right)\right)\right]$$

Therefore, we have, $(S_{\mathfrak{y}}\cup\mathbb{P}_{\mathfrak{y}})\blacksquare O_{\mathfrak{y}}\neq(S_{\mathfrak{y}}\blacksquare\mathbb{P}_{\mathfrak{y}})\cup(S_{\mathfrak{y}}\blacksquare O_{\mathfrak{y}})$

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vi. Here $(S_{\mathfrak{Y}} \cap \mathbb{P}_{\mathfrak{Y}}) \blacksquare O_{\mathfrak{Y}}$, $(S_{\mathfrak{Y}} \blacksquare \mathbb{P}_{\mathfrak{Y}}) \cap (S_{\mathfrak{Y}} \blacksquare O_{\mathfrak{Y}}) \in PNSHM$ *with order* $\partial \times \mathcal{E}$. Then

$$(S_{\mathfrak{Y}} \cap \mathbb{P}_{\mathfrak{Y}}) \blacksquare O_{\mathfrak{Y}} = \left[\left(\min(T^s_{stn}, T^p_{stn}), \frac{(I^s_{stn}, I^p_{stn})}{2}, max(F^s_{stn}, F^p_{stn})\right)\right] \blacksquare [(T^o_{stn}, I^o_{stn}, F^o_{stn})]$$

$$= \left[\left(\frac{\min(T^s_{stn}, T^p_{stn}).2T^o_{stn}}{\min(T^s_{stn}, T^p_{stn}) + T^o_{stn}}, \frac{\frac{(I^s_{stn}, I^p_{stn})}{2}.2I^o_{stn}}{\frac{(I^s_{stn}, I^p_{stn})}{2} + I^o_{stn}}, \frac{max(F^s_{stn}, F^p_{stn}).2F^o_{stn}}{max(F^s_{stn}, F^p_{stn}) + F^o_{stn}}\right)\right]$$

$(S_{\mathfrak{Y}} \cap \mathbb{P}_{\mathfrak{Y}}) \blacksquare O_{\mathfrak{Y}}$

$$= \left[\left(min\left(\frac{2T^s_{stn}.T^o_{stn}}{T^s_{stn} + T^o_{stn}}, \frac{2T^p_{stn}.T^o_{stn}}{T^p_{stn} + T^o_{stn}}\right), \left(\frac{2I^s_{stn}.I^o_{stn}}{I^s_{stn} + I^o_{stn}}, \frac{2I^p_{stn}.I^o_{stn}}{I^p_{stn} + I^o_{stn}}\right), max\left(\frac{2F^s_{stn}.F^o_{stn}}{F^s_{stn} + F^o_{stn}}, \frac{2F^p_{stn}.F^o_{stn}}{F^p_{stn} + F^o_{stn}}\right)\right)\right]$$

$(S_{\mathfrak{Y}} \blacksquare \mathbb{P}_{\mathfrak{Y}}) \cap (S_{\mathfrak{Y}} \blacksquare O_{\mathfrak{Y}})$

$$= \left[\left(min\left(\frac{2T^s_{stn}.T^o_{stn}}{T^s_{stn} + T^o_{stn}}, \frac{2T^p_{stn}.T^o_{stn}}{T^p_{stn} + T^o_{stn}}\right), \frac{\left(\frac{2I^s_{stn}.I^o_{stn}}{I^s_{stn} + I^o_{stn}}, \frac{2I^p_{stn}.I^o_{stn}}{I^p_{stn} + I^o_{stn}}\right)}{2}, max\left(\frac{2F^s_{stn}.F^o_{stn}}{F^s_{stn} + F^o_{stn}}, \frac{2F^p_{stn}.F^o_{stn}}{F^p_{stn} + F^o_{stn}}\right)\right)\right]$$

Therefore, we have, $(S_{\mathfrak{Y}} \cap \mathbb{P}_{\mathfrak{Y}}) \blacksquare O_{\mathfrak{Y}} \neq (S_{\mathfrak{Y}} \blacksquare \mathbb{P}_{\mathfrak{Y}}) \cap (S_{\mathfrak{Y}} \blacksquare O_{\mathfrak{Y}})$

**Proposition 4.2**

Let $S\mathfrak{Y} = [s_{st}]$, where $s_{st} = (T^s_{stn}, I^s_{stn}, F^s_{stn}) \in PNSHM$ *with order* $\partial \times \mathcal{E}$. Then

i. $S_{\mathfrak{Y}} \otimes^{w} S_{\mathfrak{Y}} = S_{\mathfrak{Y}}$

ii. $S_{\mathfrak{Y}} \boxtimes^{w} S_{\mathfrak{Y}} = S_{\mathfrak{Y}}$

iii. $S_{\mathfrak{Y}} \blacksquare^{w} S_{\mathfrak{Y}} = S_{\mathfrak{Y}}$

Proof: For all $s$, $t$, $n$ and $w_1, w_2 > 0$ we have

i. $$S_{\mathfrak{Y}} \otimes^{w} S_{\mathfrak{Y}} = \left[\left(\frac{w_1 T^s_{stn} + w_2 T^s_{stn}}{w_1 + w_2}, \frac{w_1 I^s_{stn} + w_2 I^s_{stn}}{w_1 + w_2}, \frac{w_1 F^s_{stn} + w_2 F^s_{stn}}{w_1 + w_2}\right)\right] = [(T^s_{stn}, I^s_{stn}, F^s_{stn})] = S_{\mathfrak{Y}}$$

ii. $$S_{\mathfrak{Y}} \boxtimes^{w} S_{\mathfrak{Y}} = \left[\begin{pmatrix} (w_1 + w_2)\sqrt{(T^s_{stn})^{w_1}.(T^s_{stn})^{w_2}}, \\ (w_1 + w_2)\sqrt{(I^s_{stn})^{w_1}.(I^s_{stn})^{w_2}}, \\ (w_1 + w_2)\sqrt{(F^s_{stn})^{w_1}.(F^s_{stn})^{w_2}} \end{pmatrix}\right] = [(T^s_{stn}, I^s_{stn}, F^s_{stn})] = S_{\mathfrak{Y}}$$

iii. $$S_{\mathfrak{Y}} \blacksquare^{w} S_{\mathfrak{Y}} = \left[\left(\frac{w_1 + w_2}{\frac{w_1}{T^s_{stn}} + \frac{w_2}{T^s_{stn}}}, \frac{w_1 + w_2}{\frac{w_1}{I^s_{stn}} + \frac{w_2}{I^s_{stn}}}, \frac{w_1 + w_2}{\frac{w_1}{F^s_{stn}} + \frac{w_2}{F^s_{stn}}}\right)\right] = [(T^s_{stn}, I^s_{stn}, F^s_{stn})] = S_{\mathfrak{Y}}$$

## 5. USING PNSHM TO SOLVE A DECISION-MAKING PROBLEM

The PNSHM-algorithm, a efficient methodology for solving Pythagorean Neutrosophic Super Hypersoft set-based decision-making problems, can be developed using matrix operations.

**Definition**

Let $S = [s_{st}]_{\partial \times \mathcal{E}}$ be a Pythagorean Neutrosophic Super Hypersoft matrix where $s_{st} = (T^s_{stn}, I^s_{stn}, F^s_{stn})$

1. if an indeterminacy membership degree (I) lies in Favor of truth-membership degree(T) then the Grace matrix of the matrix $S$ which is symbolized by $G(S)$ and it is defined as;

$$\mathbb{G}(S) = [\mathfrak{g}^s_{st}]_{\partial \times \mathcal{E}}, \text{ where } \mathfrak{g}^s_{st} = (T^s_{stn} + I^s_{stn}) - F^s_{stn}, \forall s, t, n$$

2. from the Grace matrix $G(S)$ and value matrix $V(S)$, the Mean matrix $M(S)$ is defined as;

$$\mathbb{M}(S) = [m^s_{st}]_{\partial \times \mathcal{E}} = \frac{\mathbb{V}(S) + \mathbb{G}(S)}{2}.$$

3. The Total mean of an object is given by; $\sum_{t=1}^{k} m^s_{st}$, $\forall s$ where $m^s_{st}$ are entries in the mean matrix.

## 6. THE CHARACTERISTICS OF MEAN FUNCTIONS.

Every characteristic of the real matrix is fulfilled by both the Value matrix and the Grace matrix. Consequently, the Mean matrix, derived from the Value matrix and the Grace matrix, is also a real matrix. Therefore, all properties associated with real matrices are upheld by the mean function.

## 7. METHODOLOGY

From a set of alternatives, the decision-maker identifies the optimal choice using selected SuperHypersoft attributes. When these attributes involve further Super-Hypersoft sub-attributes under a Pythagorean neutrosophic super hypersoft set (PNSHS) framework, preferences for each alternative are specified in PNSHS format relative to those sub-attributes. This yields a Pythagorean neutrosophic super hypersoft matrix (PNSHM) of size $\varsigma \times \nu$ with $\varsigma$ denoting alternatives and $\nu$ the aggregate superhypersoft sub-attributes. Subsequent steps derive the value matrix and grace matrix from the PNSHM, then compute the mean matrix and overall mean per alternative to facilitate ranking and final selection.

## 8. PNHSM-ALGORITHM

1) Derive the Pythagorean Neutrosophic Hypersoft set based on the selected attributes from the provided scenario.
2) Utilize the first step to create the PNSHM.
3) Calculate $\mathbb{V}(S)$ and $\mathbb{G}(S)$ from the result of 2.
4) Determine the mean matrix $\mathbb{M}(S)$.
5) Obtain the total mean matrix from the mean matrix.
6) The optimal solution would be to maximize $\sum_{t=1}^{k} m_{st}^{z}$.
7) If there is a maximum value for multiple alternatives in the Total Mean Matrix, the decision maker can choose any one alternative.

## 9. PROBLEM CONTEXT

A university wants to select the most suitable candidates to invite for interviews for a funded PhD position in Artificial Intelligence from an initial pool of seven applicants $A_1$,$A_2$, $A_3$,$A_4$, $A_5$,$A_6$, $A_7$. The decision is based on several main attributes, each decomposed into sub-attributes, reflecting a super-hypersoft parameterization of the evaluation space. Because expert judgments are uncertain and may be incomplete, each applicant–parameter tuple is assessed by a Pythagorean neutrosophic triple, and a suitable multi-attribute decision-making procedure is applied to rank the seven applicants and identify the top four (or fewer) for final shortlisting.

- Universe of alternatives: $\gamma$ = { $A_1$,$A_2$, $A_3$,$A_4$, $A_5$,$A_6$, $A_7$}.
- Main attributes and sub-attributes:
- $r1$: Academic performance with values {Good,Very good,Excellent}.
- $r2$: Research potential with values {Average,Strong}.
- $r3$: Programming skills with values {Basic,Intermediate,Advanced}.
- $r4$: Recommendation strength with values {Moderate,Strong}

$p(r_1)$ = {{Good},{Very good},{Excellent} ,{Good,Very good},{Good,Excellent}, {Very good,Excellent},{Good,Very good,Excellent},{∅}}

$p(r_2)$ = {{Average},{Strong},{Average,Strong},{Average,Strong},{∅}}

$p(r_3)$ = {{Basic},{Intermediate},{Advanced},{Basic,Intermediate},{Basic,Advanced}, {Intermediate,Advanced},{Basic,Intermediate,Advanced},{∅}}

$p(r_4)$ = {{Moderate},{Strong},{Moderate,Strong},{∅}}

Let the function be
$S: \mathrm{P}(\mathfrak{G}_1^{a}) \times \mathrm{P}(\mathfrak{G}_2^{b}) \times \mathrm{P}(\mathfrak{G}_3^{c}) \times \mathrm{P}(\mathfrak{G}_4^{d}) \to P(\gamma)$. The Pythagorean Neutrosophic Super Hypersoft Set is defined as; $S: \mathrm{P}(\mathfrak{G}_1^{a}) \times \mathrm{P}(\mathfrak{G}_2^{b}) \times \mathrm{P}(\mathfrak{G}_3^{c}) \times \mathrm{P}(\mathfrak{G}_4^{a}) \to P(\gamma)$. Let us assume ({Good,Very good},{Strong},{Intermediate,Advanced},{Moderate}) is the actual requirement of the university. on the basis four candidates are shortlisted $A_1$, $A_2$, $A_5$, $A_7$.

Take illustrative parameters, each alternative in the form of Pythagorean Neutrosophic Super Hypersoft Set as follows,

$$S = S(\{\text{Good,Very good}\}, \{\text{Strong}\}, \{\text{Intermediate,Advanced}\}, \{\text{Moderate}\})$$

$$= \begin{Bmatrix} (A_1, & (0.3,0.1,0.5) & (0.3,0.3,0.4) & (0.2,0.4,0.3) & (0.2,0.1,0.4)) \\ (A_2, & (0.3,0.4,0.3) & (0.5,0.2,0.3) & (0.7,0.2,0.1) & (0.2,0.2,0.5)) \\ (A_5, & (0.5,0.2,0.2) & (0.6,0.3,0.1) & (0.1,0.3,0.6) & (0.5,0.2,0.2)) \\ (A_7, & (0.4,0.3,0.3) & (0.3,0.4,0.2) & (0.2,0.1,0.1) & (0.3,0.3,0.5)) \end{Bmatrix}$$

The PNSHM derived from the above PNSH set is,

$$S = \begin{pmatrix} (0.3,0.1,0.5) & (0.3,0.3,0.4) & (0.2,0.4,0.3) & (0.2,0.1,0.4) \\ (0.3,0.4,0.3) & (0.5,0.2,0.3) & (0.7,0.2,0.1) & (0.2,0.2,0.5) \\ (0.5,0.2,0.2) & (0.6,0.3,0.1) & (0.1,0.3,0.6) & (0.5,0.2,0.2) \\ (0.4,0.3,0.3) & (0.3,0.4,0.2) & (0.2,0.1,0.1) & (0.3,0.3,0.5) \end{pmatrix}$$

The value matrix $\mathbb{V}(S) = [\mathbb{v}_{st}^{s}] = [T_{stn}^{S} - (I_{stn}^{S} + F_{stn}^{S})]$

$$\mathbb{V}(S) = \begin{pmatrix} -0.3 & -0.4 & -0.5 & -0.3 \\ -0.4 & 0 & 0.4 & -0.5 \\ 0.1 & 0.2 & -0.8 & 0.1 \\ -0.2 & -0.3 & 0 & -0.5 \end{pmatrix}$$

The grace matrix $\mathbb{G}(S) = [\mathbb{g}_{st}^{s}] = [(T_{stn}^{S} + I_{stn}^{S}) - F_{stn}^{S}]$

$$= \begin{pmatrix} -0.1 & 0.2 & 0.3 & -0.1 \\ 0.4 & 0.4 & 0.8 & -0.1 \\ 0.5 & 0.8 & -0.2 & 0.5 \\ 0.4 & 0.5 & 0.2 & 0.1 \end{pmatrix}$$

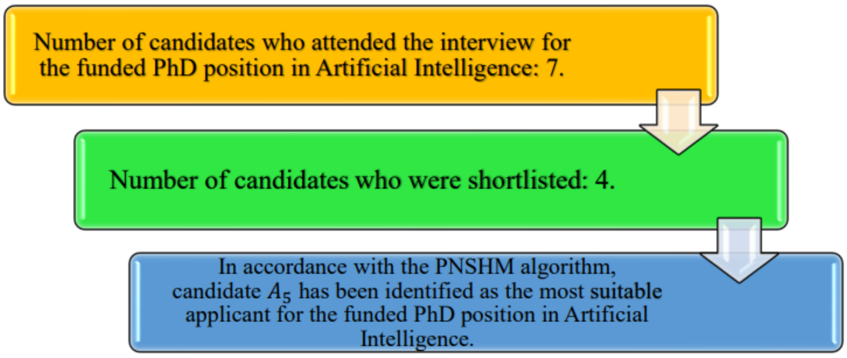
The mean matrix is $\mathbb{M}(S) = \frac{\mathbb{V}(S)+\mathbb{G}(S)}{2}$ is,

$$\mathbb{M}(S) = \begin{pmatrix} -0.2 & -0.1 & -0.1 & -0.2 \\ 0 & 0.2 & 0.6 & -0.3 \\ 0.3 & 0.5 & -0.5 & 0.3 \\ 0.1 & 0.1 & 0.1 & -0.2 \end{pmatrix}$$

The tola mean matrix $\mathbb{M}(S) = \sum_{t=1}^{k} m_{st}^{s}$

$$\mathbb{M}(S) = \begin{pmatrix} -0.6 \\ 0.5 \\ 0.6 \\ 0.1 \end{pmatrix}$$

Based on the above mean matrix, the maximum mean value is, $\max_{1 \le i \le 4}\{m_{ij}^{z}\} = A_5 = 0.6.$ therefore according to the PNSHM algorithm, candidate $A_5$ is selected for a funded Ph.D. position in Artificial Intelligence.




## 10. DISCUSSION

The proposed PNSH matrix-based methodology produces more flexible and information-preserving results than existing neutrosophic matrix approaches in multi-attribute decision-making. By encoding truth, indeterminacy, and falsity degrees for refined super-hypersoft sub-attribute combinations directly in matrix form, the approach captures intricate interactions between criteria that are usually aggregated away in conventional decision matrices, thereby substantially reducing information loss. Unlike many traditional matrix theories in which the score of one parameter does not significantly affect related entries, the structure allows parameter scores to influence each other through the underlying Pythagorean neutrosophic super hypersoft representation, so both discrimination and similarity between alternatives are reflected more faithfully in the resulting score and ranking matrices. Consequently, the PNSHM framework helps to avoid decisions based on marginal or distorted single-criterion effects and offers a robust tool for deriving reliable rankings in complex decision-making problems under severe uncertainty.

## 11. CONCLUSION

In conclusion, this study introduces key algebraic structures and operations for Pythagorean neutrosophic super hypersoft matrices (PNSHMs), validated through examples, and proposes a flexible decision-making algorithm using value, grace, and mean matrices that preserves information better than existing neutrosophic approaches. The methodology excels in handling multi-layered uncertainty for precise rankings, with potential future extensions to interval-valued or hybrid models.

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