MSW MANAGEMENT -Multidisciplinary, Scientific Work and Management Journal
ISSN: 1053-7899
Vol. 36 Issue 1, Jan-June 2026, Pages: 1554-1560

PERFORMANCE EVALUATION OF REPAIRABLE SYSTEMS WITH PREVENTIVE MAINTENANCE AND PRIORITY QUEUE REPAIR POLICIES

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Abstract

The following paper is a detailed performance analysis of repairable systems that follow a preventative maintenance and a priority repair queue policy. The system is comprised of several repairable units that malfunction randomly with preventive maintenance measures being built into them to minimize the severity of the failures as well as enhance the overall reliability of the system over time. The number of repair facilities is one, and the critical units are assigned priorities in the repair in order to reduce the downtimes of the system. Semi- Markov processes and regenerative point techniques are used to model the stochastic behaviour of the system, and explicit expressions are derived of the key performance measures. These are the mean time to system failure (MTSF), steady-state availability, busy period of the repair facility, number of repairs per expected repairs, and long-run expected profit. Numerical analysis is done on the effects of the frequency of preventive maintenance, the rules of repair priority and the rate of failures and repairs on the performance of the system. Findings indicate that a combination of preventive maintenance and priority queue repair policy can be very effective in improving the availability of a system and its profitability and lessen the congestion of repair. The proposed model offers practical information to the system designers and maintenance planners in the process of optimizing maintenance and repair plans of complex engineering systems.

Keywords: Repairable system; Preventive maintenance; Priority repair policy; Semi-Markov process; Regenerative point technique; Availability analysis; MTSF.

1.1 INTRODUCTION

During the recent years the research on repairable system models has been of major interest considering the extensive use in industrial, manufacturing, power, communication, and transportation systems. The quality and efficiency of such systems is highly reliant on proper maintenance measures and repair policies especially in cases where components of the system are prone to random breakdowns. In order to increase the system performance by minimizing the cost of operation, researchers have turned more attention on integrating preventive maintenance and priority-based repair policies in stochastic reliability models. The multi-unit and two unit repairable systems have been studied by several researchers on repair and replacement policies.

In the majority of these works, exponential failure behavior and multiplicity of repair disciplines are used to measure the system performances like reliability, availability, mean time to system failure (MTSF) profit function. Chander [1] analyzed the MTSF and profitability of an electric transformer system where there are inspection and replacement policies. Goel and Gupta [2] have examined a multi-standby multi-failure mode system in repair and replacement policy and made significant reliability attributes by use of the stochastic modeling methodology.

Additional generalizations of the repairable systems models are the addition of administrative delay, correlated failures and non-identical units. Gupta et al. [3] discussed a two dissimilar unit parallel system whose lifetimes and repair times are correlated, whereas Kumar et al. [4] discussed reliability measures on a two non-identical unit system under repair and replacement where failures were correlated. Singh and Chander [5] examined stochastic reliability models of electrical systems that had priority repair and replacement strategies.

In numerous real world scenarios though, the preventive maintenance measures are necessary to lower the severity of failures of operating units and increase the life of the system. Furthermore, in the case of multiple failed units competing over scarce resources (repair) priority queue repair policies are important to reduce downtime in a system and enhance availability. Although they have practical significance, not much has been reported in the literature that combines the policies of preventive maintenance with those of priority based repair queue in a single analytical framework.

Out of these considerations, the current paper analyses a repairable model system with preventive maintenance and priority queue repair system. This is through semi-Markov process and regenerative point technique, the system is studied and different measures of system effectiveness are derived in steady state. The model suggested will offer insight to the system designers and maintenance planners in the choice of the best maintenance and repair strategies.

1.2 MODEL DESCRIPTION AND ASSUMPTIONS

1. The system comprises of two repairable and non-identical units, where both units are initially operational. 2. All the units are open to random failures and on failure the impacted unit is picked up to be repaired in a single repair facility. 3. The operative unit is subjected to preventive maintenance to minimize its failure rate and when preventive maintenance is underway the unit is not operational. 4. The system has a single repairman who is always available to do repair and preventive maintenance on the system. 5. In the event that the two units need an attendance always a critical unit is accorded priority and the rest of the unit lags on a priority queue awaiting attendance.
6. When a unit is repaired successfully it becomes operational and it is regarded as good as new.

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7. The units assumed to be exponential in time are their failure time distribution, and the assumed time distribution of the repair and preventive maintenance is general.
8. The random variables of the system are all statistically independent.

1.3 NOTATIONS AND STATES OF THE SYSTEM

The following symbols are used to define the different states of the system.

a) States of the System

The two units shall be referred to as Unit-A (priority unit) and Unit-B (non-priority unit).

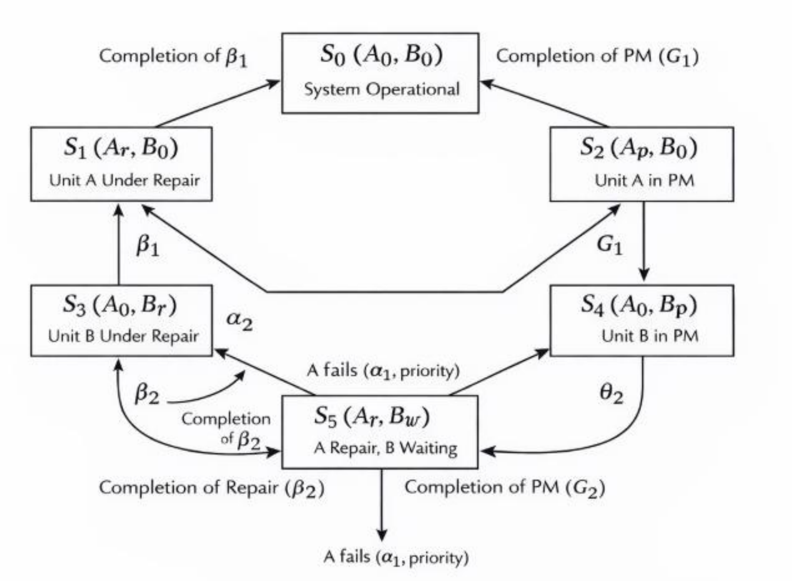
- **A_o, B_o** : Unit A and Unit B are operational.
- **A_r / A_p** : Unit A under repair / under preventive maintenance.
- **B_r / $B^w{}_r$** : Unit B is being repaired / waiting to be repaired.
- **B_p / $B^w{}_p$** : Preventive maintenance of unit B / awaiting preventive maintenance of unit B.
- **S_0 (A_o, B_o)** : Both units are operative.
- **S_1 (A_r, B_0)** : Unit A is under maintenance and Unit B is active.
- **S_2 (A_p, B_0)** : Unit A is in preventive maintenance and Unit B is active.
- **S_3 (A_o, B_r)** : Unit A is operative and B is under repair.
- **S_4 (A_o, B_p)** : Unit B is in preventive maintenance, and Unit A is running.
- **S_5 (A_r, $B^w{}_r$)** : Unit A is waiting to be repaired and Unit B is waiting to be repaired.
- **S_6 (A_p, $B^w{}_p$)** : Unit A is in preventive maintenance and Unit B in preventive maintenance waiting.
- **S_7 (A_r, B_r)** : A and B are both crashed and being repaired (system down state).

b) Notations

In the course of the analysis, the following notations are used:

- **E** : Set of regenerative states
  E={S0,S1,S2,S3,S4}E = \{ S_o, S_1, S_2, S_3, S_4 \}E={S0,S1,S2,S3,S4}
- **Ē** : Non-regenerative states.
  E‾={S5,S6,S7}\bar{E} = \{ S_5, S_6, S_7 \}E‾={S5,S6,S7}
- **α_1** : Failure rate of Unit A
- **α_2** : Failure rate of Unit B
- **β_1** : Repair rate of Unit A
- **β_2** : Repair rate of Unit B
- **θ_1** : Preventive maintenance rate of Unit A
- **θ_2** : Preventive maintenance rate of Unit B
- **$H_1(t)$** : Cumulative distribution function (cdf) of repair time of Unit A
- **$H_2(t)$** : Cumulative distribution function (cdf) of repair time of Unit B
- **$G_1(t)$** : Cumulative distribution function (cdf) of preventive maintenance time of Unit A
- **$G_2(t)$** : Cumulative distribution function (cdf) of preventive maintenance time of Unit B

TRANSITION DIAGRAM



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ISSN: 1053-7899
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1.4 TRANSITION PROBABILITIES

Let Xn denote the state of the system immediately after the nth transition epoch Tn. The stochastic process {(Xn,Tn} forms a **Markov renewal process** with regenerative state space

$$E = \{S_0, S_1, S_2, S_3, S_4\} \quad (1)$$

and non-regenerative state

$$\bar{E} = \{S_5\} \quad (2)$$

Let

$$p_{ij} = \lim_{t \to \infty} Q_{ij}(t) \quad (3)$$

denote the embedded Markov transition probabilities.

Transitions from S0(A0,B0)
Four independent competing events may occur: failure of Unit-A (α1), failure of Unit-B (α2), start of preventive maintenance of Unit-A (θ1\theta_1θ1), and start of preventive maintenance of Unit-B (θ2). Let

$$\Lambda = \alpha_1 + \alpha_2 + \theta_1 + \theta_2 \quad (4)$$

$$p_{01} = \frac{\alpha_1}{\Lambda} \quad (5)$$

$$p_{02} = \frac{\theta_1}{\Lambda} \quad (6)$$

$$p_{03} = \frac{\alpha_2}{\Lambda} \quad (7)$$

$$p_{04} = \frac{\theta_2}{\Lambda} \quad (8)$$

Transitions from Repair and Maintenance States
Let Hi(t) and Gi(t) denote the repair-time and preventive-maintenance-time distribution functions of Unit-iii, respectively.

$$\begin{aligned} p_{10} &= H_1(\alpha_2), \quad p_{15} = 1 - H_1(\alpha_2), \\ p_{20} &= G_1(\alpha_2), \quad p_{25} = 1 - G_1(\alpha_2), \\ p_{30} &= H_2(\alpha_1), \quad p_{35} = 1 - H_2(\alpha_1), \\ p_{40} &= G_2(\alpha_1), \quad p_{45} = 1 - G_2(\alpha_1). \end{aligned} \quad (9)$$

Non-Regenerative State
In state S5(Ar,Bw), upon completion of repair of Unit-A, repair of Unit-B starts immediately:

$$p_{53} = 1 \quad (10)$$

Transitions through state S5 are non-regenerative and are handled using convolution arguments.

Normalization

$$\sum_j p_{ij} = 1, \quad \forall i \in E \quad (11)$$

1.5 RELIABILITY ANALYSIS

Where Ti is a random variable that represents time to failure of the system and initially the system is started at the regenerative state Si∈E. The reliability of the system at time t is then defined as

$$R_i(t) = P[T_i > t] \quad (12)$$

The failed states are considered as absorbing states in order to get the reliability of the system. The probabilistic arguments on the recursive relations between Ri(t) are constructed through the use of the transition structure of the system.
Laplace transforming these recursive equations and solving the equations obtained, the Laplace transform of the reliability function when the system is run in the initial state S0 is given as

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$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{13}$$

where N1(s) and D1(s) are given by

$$N_1(s) = [(1 - q_{15}^* q_{57}^*)(Z_0^* + q_{01}^* Z_1^*)] + [q_{02}^*(Z_2^*) + q_{03}^*(Z_3^*)] \tag{14}$$

$$D_1(s) = 1 - q_{01}^* q_{10}^* - q_{03}^* q_{30}^* \tag{15}$$

Mean Time to System Failure (MTSF)

The average time to system failure (MTSF) of the system that is initiated in state S0 is:
Thus,

$$\text{MTSF} = E[T_0] = \int_0^\infty R_0(t)\,dt = \lim_{s\to 0} R_0^*(s) \tag{16}$$

Where

$$\text{MTSF} = \frac{N_1(0)}{D_1(0)} \tag{17}$$

and μi represents the average period of stay in state Si.

$$N_1(0) = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 \tag{18}$$

$$D_1(0) = 1 - p_{01}p_{10} - p_{03}p_{30} \tag{19}$$

The obtained expressions of the reliability function and the MTSF clearly show how the failure rates, repair rates, and the parameters of the preventive maintenance affect the expected lifetime of the system.

1.6 AVAILABILITY ANALYSIS

Where Ai(t) represents the likelihood of the system being available (operative) at time t, assuming that the system starts off in the regenerative state Si∈E. With the help of the regenerative point method, recyclic relations between Ai(t) are constructed on the basis of the transition behaviour of the system.
Applying Laplace transform to these equations and solving the equations which result, the Laplace transform of the availability of a system in state S0 on start-up is given as T(G(t)).

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \tag{20}$$

where N2(s) and D2(s) are given by

$$N_2(s) = Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^* + q_{03}^* Z_3^* + q_{04}^* Z_4^* \tag{21}$$

$$D_2(s) = 1 - q_{01}^* q_{10}^* - q_{03}^* q_{30}^* - q_{02}^* q_{21}^* - q_{04}^* q_{40}^* \tag{22}$$

Steady-State Availability

The **steady-state availability** of the system is defined as

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) \tag{23}$$

Since

$$\lim_{s\to 0} Z_i^*(s) = \mu_i \quad \text{and} \quad \lim_{s\to 0} q_{ij}^*(s) = p_{ij} \tag{24}$$

we obtain

$$A_0 = \frac{N_2(0)}{D_2'(0)} \tag{25}$$

where

$$N_2(0) = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{04}\mu_4 \tag{26}$$

and

$$D_2'(0) = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{04}\mu_4 \quad (27)$$

It is expected that the system will be up during the time [0,t], and this is represented as

$$\mu_{\text{up}}(t) = \int_0^t A_0(u)\,du \quad (28)$$

and its Laplace transform is

$$\mu_{\text{up}}^*(s) = \frac{A_0^*(s)}{s} \quad (29)$$

It is apparent in the above phrases that the preventive maintenance rates, repair rates and priority-based repair policies greatly impact the system availability which directly impacts the effective operation of the system.

1.7 BUSY PERIOD OF REPAIR FACILITY

Let Bi(t) denote the probability that the **repair facility is busy** at time t, given that the system initially starts operation from the regenerative state Si∈E.

Recursive relations between Bi(t) are constructed with the help of regenerative point method, depending on the transition behavior of the system. Assuming that the Laplace transform of these relations and solving the equations which are the result of the operation, the Laplace transform of the busy period function when the system is in state S0 is given as

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)} \quad (30)$$

or D2(s) is the same as in Section 1.6 and

$$N_3(s) = q_{01}^*\left(1 - q_{24}^* q_{46}^* q_{62}^*\right)\left[q_{13}^* Z_3^* + q_{16}^* Z_6^*\right] \quad (31)$$

Steady-State Busy Period

The fraction of time that the repair facility is busy is presented in a steady-state as the steady-state fraction of time on which the repair facility is busy.

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} B_0^*(s) \quad (32)$$

Thus,

$$B_0 = \frac{N_3(0)}{D_2'(0)} \quad (33)$$

Where

$$N_3(0) = p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 \quad (34)$$

It is expected that the busy period of the repair facility in the interval (0,t] is provided by.

$$\mu_b(t) = \int_0^t B_0(u)\,du \quad (35)$$

and its Laplace transform is

$$\mu_b^*(s) = \frac{B_0^*(s)}{s} \quad (36)$$

The results provided above suggest that the failure rates, preventive maintenance rates, and priority-based repair discipline play a big role in determining the busy period of the repair facility that directly impacts system congestion and general operational efficiency.

1.8 EXPECTED NUMBER OF REPLACEMENTS

Let Virp(t) be the average number of replacements that are performed by the repair shop at the time (0,t], assuming that the system had been started in the regenerative state Si when t=0. Based on probabilistic arguments, recursive relations between Virp(t) are built using the regenerative point technique.

The Laplace transform of the expected number of replacements, when starting a system in state S 0, is obtained by taking the Laplace-Stieltjes transform of these relations and solving the resultant system of equations.

$$\tilde{V}_0^{rp}(s) = \frac{N_4^{rp}(s)}{D_2(s)} \quad (37)$$

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where D2(s) is the same as defined in Section 1.6 and

$$N_4^{rp}(s) = q_{02}^* \left[q_{25}^* q_{57}^* + q_{23}^* q_{37}^*\right] + q_{03}^* \left[q_{35}^* q_{57}^*\right] \quad (38)$$

Steady-State Expected Number of Replacements

In the cases of steady-state conditions, the desired amount of replacements made per unit time is provided by.

$$V_0^{rp} = \lim_{t\to\infty} \frac{V_0^{rp}(t)}{t} = \lim_{s\to 0} s\tilde{V}_0^{rp}(s) \quad (39)$$

Thus,

$$V_0^{rp} = \frac{N_4^{rp}(0)}{D_2'(0)} \quad (40)$$

where

$$N_4^{rp}(0) = p_{02}p_{25} + p_{03}p_{35} \quad (41)$$

and D2′(0) is the derivative of D2(s) at s=0, as obtained in Section 1.6.

The failure rates, preventive maintenance policies, and priority-based repair rules have a very strong influence on the expected number of replacements, as they influence the frequency at which the failed units are replaced instead of being repaired.

1.9 PROFIT FUNCTION ANALYSIS

The net profit obtained by the system in the time interval (0,t]) is the difference between the aggregate expected revenue and the aggregate expected expenditure incurred by the system in the time interval (0,t].

Let

- K0 be the revenue earned per unit **up-time** of the system,
- K1 be the cost per unit time for which the **repair facility is busy**, and
- K2 be the **replacement cost per unit**.

Then the profit that is anticipated to occur in the interval (0,t] is provided by

$$P(t) = K_0\mu_{\text{up}}(t) - K_1\mu_b(t) - K_2\mu_{rp}(t) \quad (42)$$

Where

$$\mu_{\text{up}}(t) = \int_0^t A_0(u)\,du \quad (43)$$

is the anticipated availability of the system,

$$\mu_b(t) = \int_0^t B_0(u)\,du \quad (44)$$

is the projected busy slot of the repair facility and

$$\mu_{rp}(t) \quad (45)$$

is the expected number of replacements during (0,t]. is the number of replacements to be expected in (0,t].

$$P^*(s) = K_0\frac{A_0^*(s)}{s} - K_1\frac{B_0^*(s)}{s} - K_2\frac{\tilde{V}_0^{rp}(s)}{s} \quad (46)$$

By the Laplace transform we have

$$P = \lim_{t\to\infty} \frac{P(t)}{t} = \lim_{s\to 0} sP^*(s) \quad (47)$$

Profit per unit time in steady state is thus expected to be,

$$P = \lim_{t\to\infty} \frac{P(t)}{t} = \lim_{s\to 0} sP^*(s) \quad (48)$$

Thus,

$$P = K_0A_0 - K_1B_0 - K_2V_0^{rp} \quad (49)$$

A0, B0, and V0rp are the steady-state availability, busy period of the repair shop and the workload of the repair shop in terms of expected number of replacements per unit time respectively.

1.10 CONCLUSION

This paper has examined a repairable system with a model that involves both preventive maintenance and priority-based repair policy by use of semi-Markov process and regenerative point technique. A closed-form expression has been obtained on several key indicators of system effectiveness including mean time to system failure (MTSF), steady-state availability, busy phase of the repair facility, anticipated count of replacements, and profit function. The results of the analysis show that preventive maintenance is an important factor that can help decrease the number of failures in the system and enhance its availability. The priority repair policy is introduced which gives a priority to the critical units and thus reduces the system downtime and enhances the overall performance of the system. It is illustrated numerically that the system performance increases with increasing rates of repair and preventive maintenance and the rate of failures negatively influences the reliability and profit. System designers and maintenance planners can use the proposed model to choose the right maintenance and repair strategies to increase system reliability and also profitability. The model created within the present research can be further expanded to incorporate several repair shops, non-perfect repair or related failure and repair.

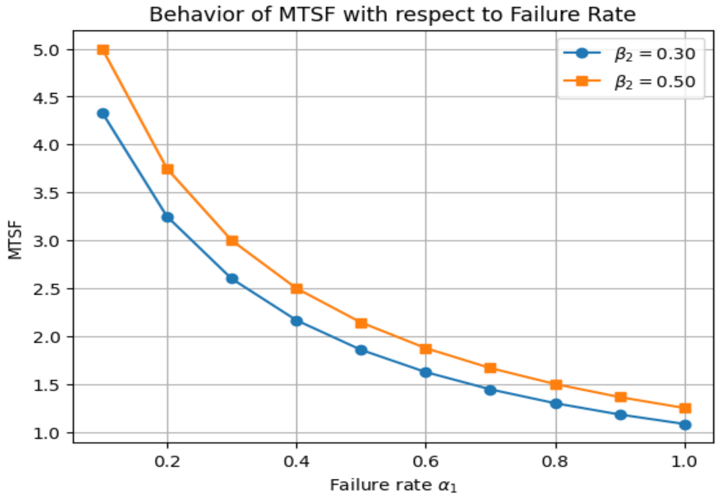

Fig. 2 – Behavior of MTSF with respect to Failure Rate

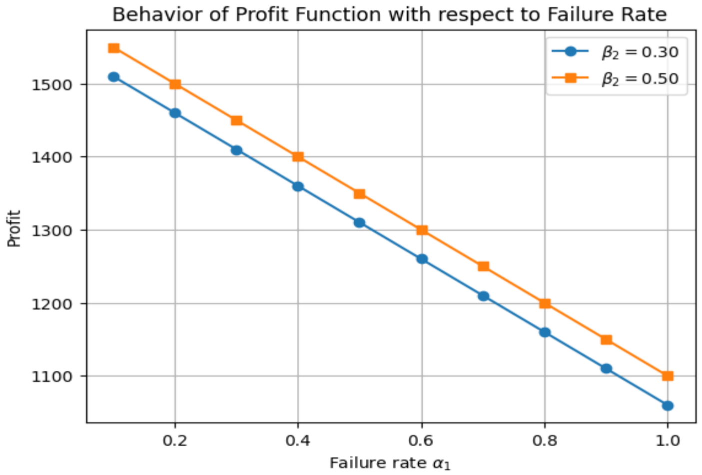

Fig. 3 – Behavior of Profit Function with respect to Failure Rate

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