

**DEVELOPMENT OF A PROBLEM-BASED LEARNING MODEL FORMULATED BASED ON THE COGNITIVE STRUCTURE OF JUNIOR HIGH SCHOOL STUDENTS**Rahma Hidayati Darwis<sup>1</sup>

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Email: [wahidah.sanusi@unm.ac.id](mailto:wahidah.sanusi@unm.ac.id)**Abstract**

Mathematical problem-solving ability is a fundamental competency that supports students' higher-order thinking skills (HOTS), yet many Indonesian junior high school students continue to struggle due to limited conceptual understanding and reliance on procedural memorization. This study aims to develop a Problem-Based Learning (PBL) model specifically formulated based on the cognitive structure of junior high school students by integrating Piaget's developmental theory and scaffolding principles. Employing a Research and Development (R&D) design adapted from the Borg and Gall model, the study involved eighth-grade students and mathematics teachers as collaborators. Data were collected through observations, interviews, questionnaires, and mathematical problem-solving tests, and analyzed using a mixed-methods approach. The developed model demonstrated high validity based on expert judgment, strong practicality according to teacher and student responses, and significant effectiveness as evidenced by improvements in students' problem-solving performance. Students showed enhanced abilities in identifying mathematical problems, formulating solution strategies, applying appropriate procedures, and evaluating results. Engagement and collaboration also increased during classroom implementation, indicating that the model successfully created a more interactive and meaningful learning environment. Scaffolding embedded in the model facilitated students' transition from concrete to abstract reasoning, aligning with their cognitive developmental stage. These findings highlight that a cognitively aligned PBL model can serve as an effective instructional innovation to strengthen mathematical problem-solving skills in junior high schools and contribute to the broader enhancement of mathematics education.

**Keywords:** Problem-Based Learning; Cognitive Structure; Mathematical Problem-Solving; Learning Model; Middle School Students

**INTRODUCTION**

Mathematical problem-solving ability is one of the essential competencies that students must acquire at both primary and secondary levels of education. This competency requires students to be able to understand information, analyze situations, and formulate appropriate and innovative strategies for solutions. In the context of modern learning, problem solving is not only regarded as a goal of mathematics education but also as a means of fostering higher-order thinking skills (HOTS), which are urgently needed in the era of the industrial revolution 4.0 and society 5.0 [1].

However, numerous studies have shown that Indonesian junior high school students' mathematical problem-solving ability remains relatively low. [2,3] found that most students only master simple procedural steps without fully understanding the concepts underlying problem solving. As a result, they struggle when confronted with unfamiliar problems. Similarly, [4] revealed that many students still perceive mathematics merely as a set of formulas to be memorized, rather than as a tool for critical and creative thinking. This finding is reinforced by [5], who reported that students' tendency to rely on mechanical thinking often leads them to give up when faced with problems requiring deeper reasoning.

This situation is also reflected in the results of the Programme for International Student Assessment (PISA) 2022, which showed that Indonesian students' mathematics performance is still below the OECD average. Such findings indicate the urgent need for innovation in mathematics instruction to prepare a generation with stronger global competitiveness. The low achievement also points to a gap between the expectations of the curriculum and classroom teaching practices.

In addition to international assessments, field observations highlight fundamental weaknesses in mastering algebraic concepts. Common errors, such as misinterpreting variable operations ( $x + x = x^2$ ) or mistakes in algebraic multiplication ( $x \cdot x = 2x$ ), suggest that students do not yet fully grasp the basic principles of symbolic manipulation. These difficulties persist when students are asked to solve equations with two variables, where they often fail to establish meaningful connections among concepts. Their tendency to present answers in an illogical or incoherent manner further illustrates limitations in mathematical reasoning, ranging from problem analysis and strategy formulation to evaluating results. Such conditions reveal that the quality of students' mathematical problem-solving ability is still far from satisfactory [6].

Teachers, as the spearhead of classroom practice, recognize the decisive role of innovative instructional models. Various approaches have been developed to improve student engagement, including Project-Based Learning (PjBL), Discovery Learning, Cooperative Learning, and Problem-Based Learning (PBL). Among these, PBL is considered

particularly promising in developing students' critical thinking, collaboration, and problem-solving skills. Many studies have confirmed the effectiveness of PBL in fostering higher-order thinking skills [7-10]

PBL has also proven to be more effective than conventional methods because it presents authentic problems that encourage students to learn actively through inquiry and group discussion. [11] showed that PBL helps students develop deeper and more meaningful understanding. Interactive learning not only enables students to master concepts but also equips them with social and communication skills. Research by [7,12] further emphasized that problem-based learning linked to real-world contexts enhances students' ability to apply mathematical knowledge meaningfully and improves both performance and attitudes toward mathematics.

Nevertheless, the implementation of PBL at the junior high school level does not always run as expected. One of the main challenges is the lack of adjustment to students' cognitive developmental stages. According to Piaget's theory, junior high school students are generally at the concrete operational stage, where their thinking tends to be logical but still heavily reliant on concrete objects and real-world experiences [13]. At this stage, students understand concepts more effectively through manipulation of physical objects or visual representations rather than abstract scenarios. This view is consistent with [14], who stressed the importance of aligning learning materials with the child's developmental level.

Previous research has emphasized that the effectiveness of PBL depends largely on its alignment with students' cognitive structures. [15] found that students at different cognitive levels respond differently to PBL scenarios, confirming that cognitive structure plays a moderating role in the effectiveness of problem-based learning. When problems are too abstract, junior high students often fail to understand the context, struggle to organize information, and cannot effectively reflect on their learning outcomes. These conditions show that while PBL is effective, its success largely depends on a design that accounts for the learners' characteristics.

Therefore, developing a PBL model integrated with Piaget's cognitive development theory becomes crucial. [16] showed that implementing PBL with a scientific approach effectively develops students' mathematics learning outcomes and increases engagement. The success of PBL is also influenced by students internal factors, such as cognitive readiness, motivation, and prior experience. [17] further emphasized the importance of scaffolding, or gradual support, to help students overcome challenges in solving complex problems.

From these findings, it can be concluded that there remains a research gap in the application of PBL at the junior high school level. Most prior studies have focused on the general effectiveness of PBL or specific aspects such as scenario design and teacher roles. Few studies have specifically examined how junior high students' cognitive levels can serve as the basis for designing more effective PBL models. In other words, there is a need to develop a PBL model that is explicitly formulated according to the cognitive structure of junior high students in order to address these limitations.

This study seeks to fill that gap. The PBL model developed in this research not only adopts general concepts but also redesigns each phase of learning in line with Piaget's cognitive developmental theory. The novelty of this study lies in the integration of cognitive scaffolding, in which problems are presented concretely and gradually increase in abstraction according to students' developmental progression. Through this approach, students are expected to build more meaningful understanding, develop problem-solving strategies, and strengthen higher-order thinking skills in a sustainable manner.

Thus, this research makes both practical and theoretical contributions. Practically, it offers teachers a model of instruction that can be directly applied in classrooms. Theoretically, it contributes to the development of PBL models based on the cognitive characteristics of junior high school students. Ultimately, the success of this research is expected to make a real contribution to improving the quality of mathematics education while preparing a generation that is better equipped to face global challenges.

## RESEARCH METHOD

### Research Design

This study employed a Research and Development (R and D) approach with the primary objective of producing an instructional model of Problem-Based Learning (PBL) specifically designed in alignment with the cognitive structure of junior high school students. The selection of this approach was grounded in the purpose of creating an educational innovation that is not merely theoretical but also practical and implementable within classroom settings. R and D is widely recognized as a systematic method in educational research, aiming to bridge the gap between theory and practice by generating validated products through iterative development and testing.

The research design followed the Borg and Gall model of educational research and development, which consists of ten systematic stages. However, these stages were carefully modified to suit the local context of mathematics learning at the junior high school level. The steps encompassed a preliminary study, needs analysis, product planning, development of the initial prototype, expert validation, small-scale field testing, product revision, larger-scale field trials, operational product refinement, and final dissemination. In addition to Borg and Gall's model, principles from Plomp's development cycle, Thiagarajan's 4-D model (Define, Design, Develop, and Disseminate), and ADDIE (Analysis, Design, Development, Implementation, Evaluation) were selectively integrated to provide flexibility and ensure that each stage was responsive to contextual challenges. This hybrid methodological approach ensured that the developed PBL model would be valid, practical, and effective for use in mathematics classrooms.

### Research Subjects and Setting

The subjects of this study were eighth-grade students of a public junior high school chosen purposively as they represented the developmental stage most relevant to the research objectives. According to Piaget's cognitive developmental theory, learners at this age range are generally in the concrete operational stage transitioning into the formal operational stage. This implies that while they are beginning to develop abstract reasoning skills, their understanding is still

highly dependent on concrete experiences and representations. Therefore, tailoring PBL to fit their cognitive structure is both timely and essential.

In addition to students, mathematics teachers were also involved as participants in the study. Their role was twofold: first, as respondents in interviews to provide professional perspectives on the feasibility and appropriateness of the model, and second, as practitioners who implemented the model in classroom trials. Teacher involvement was crucial, as their insights not only validated the practicality of the model but also enriched the refinement process by highlighting real-world challenges in teaching mathematics at the junior high level.

The research was conducted in a school environment that typifies the conditions of many Indonesian junior high schools, particularly with regard to common difficulties encountered in learning mathematics. The selection of this setting was deliberate, as it ensured that the findings would be relevant and transferable to a broader educational context. The chosen school had characteristics such as average achievement levels, limited exposure to innovative pedagogies, and a curriculum implementation aligned with national standards, making it a suitable testing ground for the proposed PBL model.

### **Types and Sources of Data**

This research utilized both qualitative and quantitative data to achieve a comprehensive understanding of the model's development and effectiveness.

Qualitative data were obtained from classroom observations, interviews with mathematics teachers, feedback from expert validators, and field notes taken during pilot implementations. These data provided rich descriptions of the teaching and learning process, the difficulties faced by students, the strategies adopted by teachers, and the contextual factors influencing the implementation of the PBL model. The qualitative component thus functioned to capture the nuances of the development process and to inform iterative revisions of the model.

Quantitative data were collected primarily through problem-solving tests in mathematics administered before and after the implementation of the model. These tests measured students' ability to understand mathematical problems, devise strategies for solutions, execute problem-solving procedures, and reflect on the correctness of their answers. The quantitative data served as the basis for evaluating the effectiveness of the model in enhancing students' mathematical problem-solving abilities.

The primary data sources were the students and teachers directly involved in the study. Meanwhile, secondary data sources included curriculum documents, lesson plans (RPP), previous studies on mathematics learning and PBL, as well as official educational reports. Combining these sources allowed the study to ground its findings both in empirical classroom realities and in theoretical and curricular frameworks.

### **Data Collection Techniques**

Several techniques were employed to gather data systematically and comprehensively.

Observation was conducted to monitor the teaching and learning process during the implementation of the PBL model. The observations focused on student engagement, participation in group discussions, problem-solving behavior, and the extent to which classroom activities followed the designed model. Observational data provided insights into the practicality of the model and revealed whether students were able to meaningfully engage with the learning activities.

Interviews were held with mathematics teachers to obtain deeper insights into their experiences using the PBL model. These semi-structured interviews allowed flexibility to explore teachers' perceptions of the strengths and weaknesses of the model, the challenges they faced in applying it, and their suggestions for improvement. The teacher interviews also helped validate whether the model was feasible within the real constraints of classroom practice, such as limited time, large class sizes, and varying student abilities.

Mathematical problem-solving tests served as the primary tool for collecting quantitative data. The tests were designed to reflect higher-order thinking skills, requiring students not only to recall formulas but also to apply reasoning, analyze situations, and synthesize solutions. Pre-tests were administered prior to the implementation of the model to establish baseline performance levels, while post-tests were conducted afterward to assess learning gains attributable to the intervention.

Questionnaires were distributed to both students and teachers to assess their perceptions of the practicality, attractiveness, and usability of the model. Student questionnaires focused on engagement, interest, and perceived learning benefits, while teacher questionnaires targeted aspects such as ease of use, clarity of instructions, and alignment with curricular objectives.

Documentation was collected to complement other data sources. This included teaching materials, lesson plans, students' written work, photographs of classroom activities, and researcher's notes. These documents served as supporting evidence and provided context for interpreting the results of observations and tests.

### **Research Instruments**

The instruments used in this study were carefully constructed and validated to ensure reliability and appropriateness.

The observation sheet included indicators related to students' active participation, collaborative problem-solving, and the degree to which the instructional steps followed the designed PBL framework. The interview guide was semi-structured, containing open-ended questions that allowed respondents to elaborate freely while still keeping the discussion aligned with the research focus.

The mathematical problem-solving test was designed around key competencies outlined in the curriculum, emphasizing understanding, planning, implementation, and reflection in problem-solving. The test items were reviewed and validated by mathematics education experts to guarantee content validity, ensuring they truly measured problem-

solving skills rather than rote memorization. Reliability of the test was examined through pilot testing with a small group of students.

The questionnaires consisted of Likert-scale items evaluating the practicality and usability of the model. For students, items assessed clarity of instructions, perceived relevance of problems, and engagement levels. For teachers, items assessed manageability, time efficiency, and adaptability to classroom conditions.

### **Data Analysis**

The analysis of research data employed a mixed-methods approach, integrating qualitative and quantitative techniques to provide a holistic picture of the model's development and impact.

Qualitative data from observations, interviews, and documentation were analyzed thematically. The process involved reducing raw data into manageable units, organizing these units into themes, and interpreting the themes to generate insights. For example, themes such as "student engagement," "cognitive difficulties," and "teacher support strategies" emerged from the qualitative analysis and were used to refine the instructional model.

Quantitative data from problem-solving tests were analyzed descriptively and inferentially. Descriptive statistics, including mean, percentage, and standard deviation, were calculated to summarize students' performance. Inferential analysis, specifically t-tests, was conducted to determine whether there were statistically significant improvements in students' problem-solving skills after the implementation of the model compared to their baseline scores.

To evaluate the quality of the developed model, the criteria proposed by [18] were applied: validity, practicality, and effectiveness. Validity referred to the appropriateness and coherence of the model as judged by experts. Practicality referred to the feasibility and ease of implementation as perceived by teachers and students. Effectiveness referred to the degree to which the model improved students' mathematical problem-solving performance. A model was deemed successful if it met all three criteria satisfactorily.

### **Research Procedures**

The overall procedure of the research was structured into three major phases: development, trial implementation, and evaluation.

In the development phase, the researcher conducted a needs analysis, reviewed relevant literature, and examined the existing practices of mathematics teaching in junior high schools. Based on these insights, a preliminary model of PBL was designed, incorporating scaffolding strategies aligned with the cognitive developmental stage of students. The initial prototype was then subjected to expert validation, where mathematics education specialists reviewed the content, structure, and feasibility of the model. Revisions were made according to expert feedback to improve the model's validity.

The trial implementation phase began with a small-scale pilot involving a limited number of students. This phase aimed to identify practical challenges in classroom settings and to test the functionality of instructional steps. Observations, student work, and teacher feedback were collected to guide necessary revisions. After refining the model, a larger-scale field test was conducted with more students across different classes to examine the model's effectiveness more comprehensively. Pre-tests and post-tests were administered during this phase to measure learning gains, and questionnaires were distributed to gather perceptions from both students and teachers.

The evaluation phase focused on analyzing the data from the trials to assess whether the model met the established criteria of validity, practicality, and effectiveness. The findings from both qualitative and quantitative analyses were integrated to provide a holistic evaluation. The final product was a PBL model adapted to the cognitive structure of junior high school students, supported by empirical evidence of its applicability and impact. This refined model was then prepared for dissemination, with the expectation that it could be adopted more widely in schools to enhance the quality of mathematics education.

## **RESULTS**

The findings of this study demonstrate that the implementation of a Problem-Based Learning (PBL) model designed in accordance with the cognitive structure of junior high school students had a significant positive effect on their mathematical problem-solving abilities. The model was trialed in mathematics classes focusing on the topic of plane geometry (*bangun datar*), an area previously identified as challenging for students due to its abstract concepts and frequent misconceptions.

### **Improvement in Problem-Solving Ability**

Quantitative results from pre-test and post-test comparisons revealed a marked improvement in students' performance after the implementation of the PBL model. Students displayed enhanced skills in identifying problems, formulating solution strategies, applying appropriate mathematical procedures, and evaluating the accuracy of their solutions. This aligns with Sumarmo's assertion that problem solving in mathematics is not merely about producing the correct answer but involves a systematic cognitive process that fosters critical, analytical, and creative thinking [9,19].

Furthermore, Bell [20] emphasized that effective problem solving requires students to clarify problems, generate hypotheses, test these hypotheses with relevant data, and evaluate outcomes. The present study corroborates this, as students trained under the structured phases of PBL increasingly demonstrated the ability to articulate problems clearly, propose multiple solution pathways, and justify their reasoning processes during classroom discussions. These outcomes also resonate with the observations of Olkin and Schoenfeld, who argued that well-structured problem-solving tasks must encourage multiple approaches and deeper conceptual exploration rather than rote application of formulas.

### **Enhanced Engagement and Collaboration**

The findings also highlighted a significant shift in students' engagement during the learning process. Classroom observations showed that students participated more actively in group discussions, exchanged perspectives with peers, and

took greater responsibility for their learning outcomes. This echoes [21] claim that PBL enhances students' readiness for real-world contexts by cultivating teamwork, communication, and collaborative problem-solving skills. [22] similarly reported that intensive social interaction during PBL is one of its core benefits, and this was evident in the present study where students collaboratively worked on authentic mathematical tasks

These findings further reinforce [23] perspective that problem-based learning not only strengthens students' cognitive skills but also stimulates curiosity, increases classroom activity, and allows knowledge to be transferred into everyday contexts. In the classroom trials of this study, students expressed higher levels of enjoyment and interest, perceiving mathematics as more meaningful and applicable to their daily experiences. This is consistent with prior studies by [8,9], which also found PBL to be effective in enhancing motivation and higher-order thinking skills.

### **Effectiveness in Aligning with Cognitive Development**

Another important result was the alignment of the PBL model with students' cognitive developmental stages. According to Piaget's theory, eighth-grade students are generally in the transitional phase from concrete operational to formal operational thinking. During the implementation, scaffolding was provided in the form of concrete examples and visual aids, gradually transitioning into abstract mathematical reasoning. This scaffolding supported students' assimilation and accommodation processes, allowing them to build robust conceptual understanding [13][14].

As observed in class, students initially struggled with abstract problem statements but, with guided practice, they were able to generalize learned strategies to new problem contexts. This progression confirms findings by [24], who noted that the effectiveness of PBL is highly dependent on its fit with students' cognitive structures. Similarly, [17] argued that providing structured support during problem-based learning significantly enhances students' achievement. The present study provides empirical evidence supporting these assertions: students became more adept at connecting prior knowledge with new tasks, showing both cognitive growth and improved problem-solving competence.

### **Summary of Results**

In sum, the results of this research establish that the PBL model developed in this study:

Significantly improved students' mathematical problem-solving abilities, as indicated by post-test gains and classroom performance. Increased student engagement, motivation, and collaboration during mathematics learning. Demonstrated validity and effectiveness when aligned with the cognitive developmental stage of junior high school students.

Thus, the findings substantiate previous claims that problem-based learning fosters critical thinking, creativity, and meaningful knowledge construction [9,21,23], while also extending these insights by showing how tailoring PBL to students' cognitive structures can yield more powerful learning outcomes.

## **DISCUSSION**

The findings of this study confirm that the integration of a Problem-Based Learning (PBL) model tailored to the cognitive developmental stage of junior high school students can significantly enhance mathematical learning outcomes. The improvements observed in students' problem-solving skills, engagement, and collaborative learning suggest that the developed model successfully addressed both cognitive and motivational dimensions of learning mathematics.

### **Problem-Solving as the Core of Mathematics Learning**

The results revealed that students demonstrated notable gains in their ability to identify problems, devise strategies, implement mathematical procedures, and evaluate solutions. These findings reinforce [25] assertion that mathematical problem solving extends far beyond producing the correct answer; rather, it represents a systematic cognitive journey involving critical, analytical, and creative processes. This perspective aligns with Branca [15], who positioned problem solving as the heart of mathematics education, enabling learners to face novel challenges and adapt strategies accordingly. Moreover, Bell's Bell [20] emphasis on the importance of clarifying problems, formulating hypotheses, and critically evaluating outcomes is reflected in the current study. Students engaged in the structured phases of PBL increasingly demonstrated the ability to articulate problems, propose multiple solution pathways, and justify their reasoning during classroom interactions. This corresponds with Olkin and Schoenfeld's view (as cited in [9]) that well-designed problem-solving tasks should invite diverse approaches, stimulate deeper conceptual exploration, and discourage rote memorization. The findings therefore contribute further empirical support to the argument that PBL nurtures higher-order thinking by embedding reasoning and justification into everyday classroom practices.

### **Engagement, Motivation, and Collaborative Learning**

Beyond cognitive gains, the study demonstrated that students' engagement and motivation increased substantially under the PBL model. Classroom observations revealed more dynamic group discussions, active peer collaboration, and heightened responsibility for learning outcomes. These findings validate [21] claim that PBL prepares students for real-world contexts by cultivating essential teamwork and communication skills. Similarly, [22] emphasized that the social interaction inherent in PBL contributes to deeper learning, an outcome that was clearly visible in this study.

The heightened enthusiasm expressed by students also resonates with Haudi's (2021) perspective that problem-solving approaches enrich learning by making it more attractive and relevant to students' lives. Students perceived mathematics as more meaningful and enjoyable, echoing the findings of [8,9], who reported that PBL not only strengthens higher-order thinking skills but also sustains student interest. Thus, the present research highlights how PBL, when adapted to developmental readiness, can simultaneously foster cognitive, affective, and social outcomes.

### **Cognitive Alignment and the Role of Scaffolding**

A central contribution of this research is its evidence that aligning PBL with students' cognitive developmental stages significantly enhances its effectiveness. In accordance with Piaget's theory, eighth-grade students are transitioning from concrete operational to formal operational stages, necessitating learning environments that provide both tangible supports

and opportunities for abstract reasoning [13][14]. The scaffolding embedded in the PBL model — beginning with concrete tasks and gradually progressing to abstract generalizations — enabled students to assimilate and accommodate new knowledge, thereby strengthening their conceptual structures.

This finding corroborates [24], who observed that PBL's success hinges on its congruence with learners' cognitive structures, and [17], who argued that structured guidance during PBL enhances students' achievement. The results also expand upon these studies by showing how scaffolding strategies tailored to cognitive readiness facilitated students' ability to generalize problem-solving strategies and apply them across contexts. This suggests that cognitive alignment is not merely an accessory to PBL but an essential design principle that maximizes its pedagogical potential.

### **Practicality, Validity, and Effectiveness of the Model**

The validation of the PBL model against [18] criteria further strengthens its significance. The model was judged valid by experts, practical by teachers, and effective by students. Teachers confirmed that the steps were feasible within classroom constraints, while students demonstrated both measurable performance gains and positive perceptions of learning. These results are consistent with [9], who found that well-structured PBL models significantly improve mathematical problem solving, and [26], who stressed that PBL effectiveness depends on its alignment with students' cognitive characteristics. By meeting all three criteria — validity, practicality, and effectiveness the model provides a reliable framework for improving mathematics education in junior high schools.

### **Theoretical and Practical Implications**

Theoretically, this study extends the constructivist paradigm by providing concrete evidence that tailoring instructional design to cognitive development enhances knowledge construction. The results underscore Piaget's theory that learning is most effective when instructional tasks align with learners' developmental stages, and they reinforce Vygotsky's principle of scaffolding as an essential tool in guiding learners toward higher levels of competence.

Practically, the findings offer teachers a model that is both innovative and feasible. The developed PBL model provides structured guidance without overwhelming learners, making it adaptable to varied classroom contexts. Teachers can use it to design lessons that are engaging, cognitively appropriate, and aligned with curriculum standards. By emphasizing authentic problems and collaborative inquiry, the model encourages students to perceive mathematics as meaningful and transferable to real-life situations.

### **Limitations and Future Directions**

Despite its contributions, the study has certain limitations. It was conducted in a specific school context and focused on the topic of plane geometry (*bangun datar*), which may limit generalizability. Future research should replicate the study across different mathematical domains, grade levels, and school settings to confirm the broader applicability of the model. Longitudinal studies are also needed to assess the long-term impact of the model on students' cognitive growth, motivation, and learning habits.

### **Discussion**

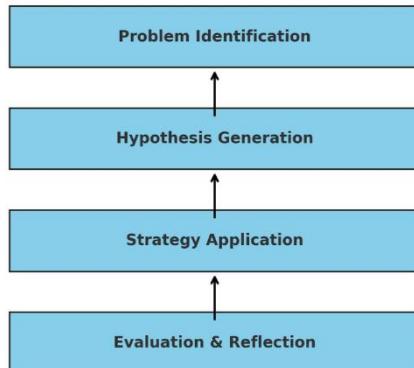
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Figure 1. Problem Solving Stages

**PBL Problem-Solving Stages**

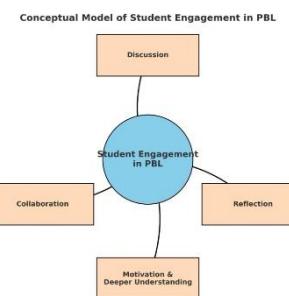


Moreover, Bell's [20] emphasis on the importance of clarifying problems, formulating hypotheses, and critically evaluating outcomes is reflected in the current study. Students engaged in the structured phases of PBL increasingly demonstrated the ability to articulate problems, propose multiple solution pathways, and justify their reasoning during classroom interactions. This corresponds with Olkin and Schoenfeld's view (as cited in [25]) that well-designed problem-solving tasks should invite diverse approaches, stimulate deeper conceptual exploration, and discourage rote memorization. The findings therefore contribute further empirical support to the argument that PBL nurtures higher-order thinking by embedding reasoning and justification into everyday classroom practices.

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Figure 2. Conceptual Model of Student



The heightened enthusiasm expressed by students also resonates with [23] perspective that problem-solving approaches enrich learning by making it more attractive and relevant to students' lives. Students perceived mathematics as more meaningful and enjoyable, echoing the findings of [8,9], who reported that PBL not only strengthens higher-order thinking skills but also sustains student interest. Thus, the present research highlights how PBL, when adapted to developmental readiness, can simultaneously foster cognitive, affective, and social outcomes.

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Figure 3. Cognitive Development

**Cognitive Development and Scaffolding (Piaget)**



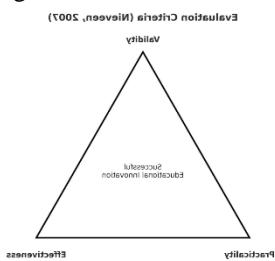
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Figure 4. Evaluation Criteria



### Theoretical and Practical Implications

Theoretically, this study extends the constructivist paradigm by providing concrete evidence that tailoring instructional design to cognitive development enhances knowledge construction. The results underscore Piaget's theory that learning is most effective when instructional tasks align with learners' developmental stages, and they reinforce Vygotsky's principle of scaffolding as an essential tool in guiding learners toward higher levels of competence.

Practically, the findings offer teachers a model that is both innovative and feasible. The developed PBL model provides structured guidance without overwhelming learners, making it adaptable to varied classroom contexts. Teachers can use it to design lessons that are engaging, cognitively appropriate, and aligned with curriculum standards. By emphasizing authentic problems and collaborative inquiry, the model encourages students to perceive mathematics as meaningful and transferable to real-life situations.

### Limitations and Future Directions

Despite its contributions, the study has certain limitations. It was conducted in a specific school context and focused on the topic of plane geometry (*bangun datar*), which may limit generalizability. Future research should replicate the study across different mathematical domains, grade levels, and school settings to confirm the broader applicability of the model. Longitudinal studies are also needed to assess the long-term impact of the model on students' cognitive growth, motivation, and learning habits.

### CONCLUSION

This study concludes that the development and implementation of a Problem-Based Learning (PBL) model designed in alignment with the cognitive structures of junior high school students significantly improved their mathematical problem-solving abilities, enhanced their engagement, and fostered collaborative learning. The model was found to be valid, practical, and effective, as confirmed through expert validation, teacher feedback, and student performance gains. The results demonstrate that PBL, when systematically structured and scaffolded to match learners' cognitive developmental stages, provides not only measurable academic benefits but also broader impacts on students' motivation and readiness to apply mathematical knowledge in real-life contexts. The findings reinforce the perspectives of Piaget on cognitive growth, Plomp's criteria for educational innovation, and constructivist theories of learning, all of which highlight the importance of active, meaningful, and developmentally appropriate instruction. Practically, this research provides teachers with a feasible instructional model that bridges abstract mathematical concepts with concrete experiences, thereby reducing student difficulties in mathematics learning. Theoretically, it extends the literature on PBL by showing that cognitive alignment and scaffolding are not supplementary features but essential design principles that determine the effectiveness of problem-based instruction. Overall, this study affirms that a cognitively tailored PBL model can serve as a reliable and innovative alternative in mathematics education at the junior high school level. Future studies are encouraged to replicate and adapt this model across diverse mathematical domains, school settings, and longer periods to further strengthen its applicability and long-term impact.

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**CONFLICT OF INTEREST**

The authors declare that there is no conflict of interest regarding the publication of this article. All authors contributed equally to the research process and manuscript preparation. There are no financial, professional, or personal relationships that could have influenced the work reported in this study.

**DECLARATION OF GENERATIVE AI**

The authors declare that no generative artificial intelligence (AI) or AI-assisted technologies were used in the writing, editing, data analysis, or preparation of this manuscript. All components of the research, including conceptualization, methodology development, data collection, analysis, interpretation, and manuscript writing, were conducted entirely by the authors.

**AUTHOR CONTRIBUTIONS**

Rahma Hidayati Darwis conceptualized the study, designed the methodology, and prepared the initial draft of the manuscript. Abdul Rahman contributed to data collection, data validation, and data analysis. Alimuddin supervised the overall research process, provided critical revisions, and ensured the academic quality of the manuscript. Wahidah Sanusi supported the interpretation of data, contributed to the literature review, and assisted in refining the discussion section. All authors read and approved the final version of the manuscript and agreed to its submission for publication.

**ETHICS APPROVAL**

This study was conducted in accordance with established ethical standards for research involving human participants. Ethical approval was granted by the Ethics Committee of Universitas Negeri Makassar, Indonesia. Participation of all students and teachers involved in the study was entirely voluntary, and informed consent was obtained prior to data collection. The confidentiality and anonymity of all participants were strictly maintained throughout the research process.

**DATA AVAILABILITY**

The data supporting the findings of this study, titled "*Development of a Problem-Based Learning Model Formulated Based on the Cognitive Structure of Junior High School Students*," are available from the corresponding author upon reasonable request. In accordance with ethical guidelines and to protect participant confidentiality, the raw survey and interview data cannot be publicly shared.

**ABBREVIATION**

Abbreviation	Full Form (English)
<b>ADDIE</b>	Analysis, Design, Development, Implementation, dan Evaluation
<b>PBL</b>	Problem-Based Learning
<b>HOTS</b>	Higher-Order Thinking Skills
<b>OECD</b>	Organisation for Economic Co-operation and Development
<b>R&amp;D</b>	Research and Development
<b>4-D</b>	Define, Design, Develop, dan Disseminate

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