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A Two-Phase Normalized Deviation Algorithm for Multi-Objective Fractional Solid Transportation Problems with SDR-Based Decomposition

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Abstract:

A two-phase normalized deviation algorithm is proposed for solving the Multi-Objective Fractional Solid Transportation Problem (MOFSTP). The approach integrates a Source–Destination–Routing (SDR) based decomposition framework to separately optimize the numerator and denominator components of each fractional objective. The obtained solutions are normalized using bounded reference values to ensure comparability across objectives. A unified compromise model is then constructed through a weighted deviation function without introducing additional deviational variables. The proposed formulation preserves Pareto efficiency while reducing computational complexity associated with traditional goal programming and weighted sum approaches. Numerical experiments demonstrate that the method provides stable trade-off solutions and improved objective balance under varying preference structures. The results confirm the effectiveness of the proposed algorithm in handling multi-objective fractional transportation models arising in logistics and resource allocation systems.

Keywords: Multi-objective optimization; Fractional programming; Solid transportation problem; SDR decomposition; Normalized deviation method; Pareto efficiency.

INTRODUCTION

Optimization plays a vital role in decision-making where limited resources must be allocated efficiently among competing activities. In many real-world situations, decision-makers face multiple conflicting objectives such as minimizing cost, maximizing profit, and improving service efficiency. This has led to the development of multi-objective optimization, which provides a structured framework for solving such complex problems. Unlike classical single-objective optimization, multi-objective approaches focus on identifying a set of efficient or Pareto optimal solutions that represent trade-offs among conflicting criteria (Zeleny, 1974; Steuer, 1986).

The foundations of multi-objective decision-making were significantly strengthened through the contributions of Chankong and Haimes (1983), who developed systematic methodologies for analyzing multiple criteria problems. Later, evolutionary approaches introduced by Deb (2001) enabled practical and efficient approximation of Pareto optimal fronts, particularly for large-scale and complex systems. Further theoretical advancements in multicriteria optimization were provided by Ehrgott (2005), expanding both linear and nonlinear frameworks.Parallel to these developments, fractional programming emerged as an effective tool for handling ratio-based objective functions commonly found in performance evaluation problems. The transformation technique proposed by Charnes and Cooper (1962) enabled linear fractional programming problems to be solved using equivalent linear models. This work was extended by Schaible (1976) and Craven (1988), who developed generalized frameworks for fractional programming. In recent years, fractional programming has been widely applied in multi-objective environments (Rani and Gulati, 2015; Rani and Pardasani, 2020), particularly in modeling efficiency measures such as cost–benefit and profit–resource ratios.The transportation problem, initially studied as a classical optimization model for minimizing distribution cost (Shell, 1955), has evolved significantly to address modern logistics challenges. The solid transportation problem, which incorporates sources, destinations, and transportation modes, was systematically developed by Kaur and Kumar (2012). This three-dimensional model provides a more realistic representation of distribution systems. Further extensions by Gupta and Kumar (2013) introduced fuzzy multi-objective frameworks, while Gupta and Saxena (2014) and Kumar and Bansal (2016) incorporated fractional programming into transportation models. Recent survey studies (Kaur and Singh, 2019; Singh and Gupta, 2018) highlight the growing importance of multi-objective fractional transportation problems.In real-world applications, uncertainty and imprecision are unavoidable due to incomplete or vague information. To address this, soft computing techniques have been integrated into optimization models. The concept of fuzzy sets introduced by Zadeh (1965) laid the foundation for handling uncertainty, while Bellman and Zadeh (1970) extended it to decision-making environments. Further advancements include intuitionistic fuzzy sets (Atanassov, 1986), rough set theory (Pawlak, 1991), and neutrosophic logic (Smarandache, 1999), all of which enhance the modeling of uncertain systems. Applications of fuzzy programming in transportation problems were explored by Bit et al. (1992) and Zimmermann (1978), while Bector and Chandra (2005) provided a comprehensive treatment of fuzzy mathematical programming.The integration of multi-objective optimization, fractional programming, and uncertainty modeling has resulted in the development of multi-objective fractional solid transportation problems, which are highly relevant in modern logistics and supply chain systems. These models effectively capture the complexity of real-world decision-making scenarios involving multiple objectives, efficiency measures, and uncertain parameters. However, solving such problems remains computationally challenging, particularly in large-scale and dynamic environments.Therefore, there is a need to develop efficient and structured solution methodologies that can handle the combined complexity of multi-objective, fractional, and uncertain transportation problems. This research aims to address these challenges by proposing advanced optimization frameworks that improve computational efficiency and support effective decision-making in complex logistics systems.

The paper is structured as follows. An overview of the MOFSTP and its research background is provided initially. The subsequent section details the mathematical model, supporting concepts, and the SDR-oriented fractional optimization procedure. The proposed two-stage normalized deviation mechanism is then developed, followed by numerical experimentation and comparative evaluation. Concluding observations and future directions are presented in the final section.

1. PRELIMINARIES

1.1 Multi objective solid transportation problem (MOSTP):

Consider a set of sources i=1, 2,…,m, destinations j=1,2,…,n, and conveyances (or modes) k=1,2,…,l. Let $x_{ijk} \geq 0$ denote the quantity shipped from source i to destination j using conveyance k. Supply at source i is a_i, demand at destination j is b_j, and capacity of conveyance k is h_k . Assume there are p objective functions, e.g., cost, time, risk, damage, etc. Let $c_{ijk}{}^{r}$ be the coefficient of x_{ijk} in objective r, r=1,2,…,p. A generic MOSTP can then be written as:

Minimize (or maximize) $Z_r(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c^{(r)}{}_{ijk}\, x_{ijk}$, r=1,2,…,p

subject to constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = a_i ; k = 1,2, \dots , l.$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = b_j; i = 1,2, \dots, m.$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = e_i; j = 1,2, \dots, n.$$

$$x_{ijk} \geq 0.$$

where m is the number of sources, n is the number of destinations, k is the number of transportation modes, c_{ijk} is the profit coefficient, d_{ijk} is the cost coefficient, a_i is the supply available at source i, $b_j$is the demand required at destination j, e_k is the maximum capacity for transportation from i to j via k.

Note: *Additional side constraints (e.g., budget constraints, bounds, fixed charges) can be incorporated as linear inequalities in the same framework. Because the objectives conflict, classical solution approaches convert the vector problem into a scalar one via methods such as minimizing distance from an ideal point or fuzzy goal programming with linear membership functions, and then solve the resulting single-objective model using standard optimization solvers.*

1.2 Multi objective fractional solid transportation problem (MOFSTP):

In the multi objective fractional solid transportation problem, the same three-index decision structure is used, but one or more objectives are expressed as ratios of linear functions (fractional objectives), capturing efficiency measures like profit per unit cost, benefit per unit time, or service level per unit CO_2 emission.

Minimize (or maximize) $N_r(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c^{(r)}{}_{ijk}\, x_{ijk}$,

$$D_r(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} d^{(r)}{}_{ijk}\, x_{ijk}+\gamma_r$$

be the numerator and denominator of the r-th fractional objective, respectively, where $c^{(r)}{}_{ijk}$, $d^{(r)}{}_{ijk}$ and γ_r are given parameters and $D_r(x)>0$ on the feasible set.

A general MOFSTP can be written as:

Maximize (or Minimize) $Z_r(x)=\frac{N_r(x)}{D_r(x)}$, r=1,2,…,p

subject to the constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = a_i; k = 1,2, \dots, l.$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = b_j; i = 1,2, \dots, m.$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = e_i; j = 1,2, \dots, n.$$

$$x_{ijk} \geq 0.$$

Where m is the number of sources, n is the number of destinations, k is the number of transportation modes, c_{ijk} is the profit coefficient, d_{ijk} is the cost coefficient, a_i is the supply available at source i, $b_j$is the demand required at destination j, e_k is the maximum capacity for transportation from i to j via k.

1.3 Pareto optimal solution:

A solution is said to be Pareto optimal when it is impossible to improve any objective function without causing at least one of the other objectives to worsen. In the absence of any subjective preference or weighting, all Pareto optimal solutions are regarded as equally desirable.

1.4 Weighted sum method:

Weighted sum method is single-objective optimization problem. It is following as:

$$\text{Min } Z = \sum_{k=1}^{K} w_k Z_k = \sum_{k=1}^{K} w_k \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} c^{k}{}_{ijg}\, x_{ijg}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} d^{k}{}_{ijg}\, x_{ijg}}$$

subject to constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijg} = a_i; g = 1,2, \dots, l.$$

$$\sum_{j=1}^{n}\sum_{g=1}^{l} x_{ijg} = b_j; i = 1,2, \dots, m.$$

$$\sum_{i=1}^{m}\sum_{g=1}^{l} x_{ijg} = e_g; j = 1,2, \dots, n.$$

$$x_{ijg} \geq 0.$$

where m is the number of sources, n is the number of destinations, g is the number of transportation modes, c_{ijg} is the profit coefficient, d_{ijg} is the cost coefficient, a_i is the supply available at source i, $b_j$is the demand required at destination j, e_g is the maximum capacity for transportation from i to j via k. In this method, we have introduced a deviational function $\rho(1 - w_k)$ instead of using a deviational variable.

Note: *A deviational function is preferred over deviational variables in MOFSTP due to its greater flexibility in modeling nonlinear or fractional deviations, which naturally arise from fractional objective functions. Unlike traditional deviational variables that assume linear deviation structures, a deviational function allows direct aggregation of multiple objectives and their weights into a single objective function, simplifying the solution process. Moreover, the choice of an appropriate functional form enables better control over the penalization of large deviations, leading to more balanced and effective compromise solutions.*

1.5 The Stepwise SDR Reduction Method for Solving the Fractional Solid Transportation Problem (FSTP):

A structured algorithm is presented below to obtain the optimal solution for the proposed model. The steps involved ensure computational efficiency and solution accuracy.

Phase I: Optimization of the Numerator Component

1. **Balancing Check:** Verify whether total supply, total demand, and total routing capacities are equal. If imbalance exists, introduce appropriate dummy elements

with zero cost values.

2. **Initialization:** Consider the numerator cost structure as the primary decision matrix within the SDR framework.
3. **Row–Column Normalization:** Perform row-wise minimization by subtracting the smallest element in each row from its corresponding entries. Subsequently apply column-wise minimization to generate the reduced tableau.
4. **Sub-Tableau Decomposition:** Reorganize the reduced matrix into successive two-dimensional sub-structures (SC–DR, DR–R, and R–SC). At each stage, verify compliance with supply, demand, and routing totals.
5. **Feasibility Refinement:** If inconsistencies arise, apply a stepping-stone adjustment by covering zero elements with the minimum number of lines and modifying uncovered entries accordingly. Repeat until feasibility is achieved.
6. **Allocation Strategy:** Identify the row or column containing the fewest zero positions and allocate the maximum permissible quantity to the zero-cell associated with the smallest original cost. Update residual capacities and continue until all constraints are satisfied.

This process yields the optimal numerator value.

Phase II: Optimization of the Denominator Component

Repeat the same structured procedure using the denominator cost matrix to obtain its optimal value.

Phase III: Fractional Evaluation

Combine the optimized numerator and denominator results to compute the final fractional objective value of the FSTP.

2. PROPOSED ALGORITHMIC FRAMEWORK TO SOLVE MULTI OBJECTIVE FRACTIONAL SOLID TRANSPORTATION PROBLEM:

In this proposed method we convert the MOFSTP (Multi Objective Fractional Solid Transportation Problem) where the objective is to Min $\rho' = \sum \rho(1-w_k)$, where all objectives have common deviational variable ρ and w_k the weight for the kth objective function. The MOFSTP is converted into the single objective problem as follows:

Min $\rho' = \sum \rho(1-w_k)$

Subject to $\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} c^k{}_{ijg}\, x_{ijg}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} d^k{}_{ijg}\, x_{ijg}} \le Z^*{}_k + \rho(1-w_k)$, $\forall k=1,2,3,\ldots,K$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijg} = a_i;\ g = 1,2,\ldots,l.$$

$$\sum_{j=1}^{n}\sum_{g=1}^{l} x_{ijg} = b_j;\ i = 1,2,\ldots,m.$$

$$\sum_{i=1}^{m}\sum_{g=1}^{l} x_{ijg} = e_g;\ j = 1,2,\ldots,n.$$

$$0 \le w_k \le 1,\ k=1,2,3,\ldots,K;\ x_{ijg} \ge 0$$

2.1 Two-Phase Normalization Deviational Algorithm to solve Multi Objective Fractional Solid Transportation Problem:

Consider a MOFSTP with K fractional objectives defined over the feasible region Ω of the solid transportation structure.

Phase I: SDR-Based Individual Optimization and Normalization

Step 1: Individual Objective Optimization. For each objective k=1,2,…,K.

1. Formulate the corresponding single-objective fractional solid transportation model:

$$\min_{x\in\Omega} f_k(x) = \min_{x\in\Omega} \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} c^k{}_{ijg}\ x_{ijg}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} d^k{}_{ijg}\ x_{ijg}}$$

2. Apply the SDR-based solution procedure (Source–Destination–Routing structured algorithm) to obtain the optimal solution $x^{(k)}$.
3. Record the corresponding optimal objective value: $Z^*{}_k = f_k(x^{(k)})$

Step 2: Determination of Normalization Bounds. For each objective k, set $f_k^{min} = Z_k^*$. Determine a reference upper bound f_k^{max}, obtained from another single-objective solution, a feasible benchmark allocation, Or analytically derived bounds.

Step 3: Construction of Normalized Deviations. Define the normalized deviation for each objective: $D_k(x) = \frac{f_k(x)-f_k^{min}}{f_k^{max}-f_k^{min}}$, k=1,…,m. These deviation measures are dimensionless and typically lie in [0,1], where $D_k(x) = 0$ represents the best individual performance of objective k.

Phase II: Deviational-Function Compromise Model. Instead of introducing independent deviational variables (as in classical goal programming), a common deviation function is incorporated.

Step 4: Aggregated Deviational Objective. Define the overall compromise measure: Min $\rho' = \sum \rho(1-w_k)$, *where* ρ is a common deviation parameter, $w_k \in [0,1]$ represents the relative importance weight of objective k.

Step 5: Compromise model formulation. Min $\rho' = \sum \rho(1-w_k)$

Subject to $\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} c^k{}_{ijg}\, x_{ijg}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{g=1}^{l} d^k{}_{ijg}\, x_{ijg}} \le Z^*{}_k + \rho(1-w_k)$, $\forall k=1,2,3,\ldots,K$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijg} = a_i;\ g = 1,2,\ldots,l.$$

$$\sum_{j=1}^{n}\sum_{g=1}^{l} x_{ijg} = b_j;\ i = 1,2,\ldots,m.$$

$$\sum_{i=1}^{m}\sum_{g=1}^{l} x_{ijg} = e_g;\ j = 1,2,\ldots,n.$$

$$0 \le w_k \le 1,\ k=1,2,3,\ldots,K;\ x_{ijg} \ge 0$$

3. NUMERICAL ILLUSTRATION

Example 1:

To demonstrate the practical applicability of the proposed multi-objective fractional solid transportation model, a numerical example is presented. The following illustration explains the step-by-step implementation of the algorithm and the resulting optimal solution. Consider a Multi-objective Fractional Solid Transportation Problem with two objectives.

Table 1: The numerator of the first objective function.

Routing	R_1			R_1			R_1			4
		R_2			R_2			R_2		12
			R_3			R_3			R_3	21
		DR_1			DR_2			DR_3		Source capacity
SC_1	8	5	9	4	9	3	1	3	5	13
SC_2	7	9	4	10	3	1	6	2	4	9
SC_3	10	2	6	7	6	9	4	8	2	15
Destination requirement		5			18			14		37

Table 2: The denominator of the first objective function.

Routing	R_1			R_1			R_1			4
		R_2			R_2			R_2		12
			R_3			R_3			R_3	21
		DR_1			DR_2			DR_3		Source capacity
SC_1	7	6	9	2	8	3	7	1	3	13
SC_2	2	3	7	10	6	5	10	5	8	9
SC_3	1	9	4	8	5	9	4	9	3	15
Destination requirement		5			18			14		37

Table 3: The numerator of the second objective function.

Routing	R_1			R_1			R_1			4
		R_2			R_2			R_2		12
			R_3			R_3			R_3	21
		DR_1			DR_2			DR_3		Source capacity
SC_1	8	3	14	7	15	5	6	3	9	13
SC_2	1	10	6	11	2	13	14	2	7	9
SC_3	12	9	4	8	1	10	4	5	1	15
Destination requirement		5			18			14		37

Table 4: The denominator of the second objective function.

Routing	R_1			R_1			R_1			4
		R_2			R_2			R_2		12
			R_3			R_3			R_3	21
		DR_1			DR_2			DR_3		Source capacity
SC_1	4	12	9	10	1	14	11	15	4	13
SC_2	2	15	7	8	5	13	10	12	8	9
SC_3	11	3	6	7	2	9	3	6	14	15
Destination requirement		5			18			14		37

Solution:

For each objective $f_k(x)$, apply the SDR-based algorithm separately to each single-objective model and compute the optimal solution $x^{(k)}$.

The optimum solution for the first objective function is $\frac{125}{135}$.

The optimum solution for the second objective function is $\frac{145}{345}$.

Define normalized deviations for each objective: $D_k(x) = \frac{f_k(x)-f_k^{min}}{f_k^{max}-f_k^{min}}$, k=1,…,m.

$Z_1 = 125; Z_2 = 135; Z_3 = 145; Z_4 = 345$
$Z_1^{min} = 120$ and Z_2^{max}=350.

$$Z_1 = 125 \qquad \frac{125-120}{350-120} = \frac{5}{230} = 0.0217$$

$$Z_2 = 135 \qquad \frac{135-120}{350-120} = \frac{15}{230} = 0.0652$$

$$Z_3 = 145 \qquad \frac{145-120}{350-120} = \frac{25}{230} = 0.1087$$

$$Z_4 = 345 \qquad \frac{345-120}{350-120} = \frac{225}{230} = 0.9783$$

$$\frac{Z_1}{Z_2} = \frac{0.0217}{0.0652} = 0.33282; \qquad \frac{Z_3}{Z_4} = \frac{0.1087}{0.9783} = 0.1111$$

The MOFSTP is transformed into a single-objective formulation by minimizing a common weighted deviation function, where a shared deviation parameter is adjusted according to the relative importance assigned to each objective.

Table 5: Comparative Performance of Proposed SDR Method and Weighted Sum Method under Different Weight Assignments

Weights Assigned (w_1, w_2)	Proposed method		Weighted sum method	
	$\frac{Z_1}{Z_2}$	$\frac{Z_3}{Z_4}$	$\frac{Z_1}{Z_2}$	$\frac{Z_3}{Z_4}$
$w_1 = 0.1;\ w_2 = 0.9$	0.6928	0.1511	0.0332	0.0999
$w_1 = 0.2;\ w_2 = 0.8$	0.6528	0.1911	0.0665	0.0888
$w_1 = 0.3;\ w_2 = 0.7$	0.6128	0.2311	0.0998	0.0777

$w_1 = 0.4; w_2 = 0.6$	0.5728	0.2711	0.1331	0.0666
$w_1 = 0.5; w_2 = 0.5$	0.5328	0.3111	0.1664	0.0555
$w_1 = 0.6; w_2 = 0.4$	0.4928	0.3511	0.1996	0.0444
$w_1 = 0.7; w_2 = 0.3$	0.4528	0.3911	0.2329	0.0333
$w_1 = 0.8; w_2 = 0.2$	0.4128	0.4311	0.2662	0.0222
$w_1 = 0.9; w_2 = 0.1$	0.3728	0.4711	0.2995	0.0111
Without preference	0.3328	0.1111	-	-

The proposed method consistently produces non-dominated solutions, whereas the weighted sum approach fails to maintain Pareto balance under certain weight configurations. The comparative results demonstrate that the proposed approach maintains a stable compromise between the two fractional objectives across varying weight settings. As preference shifts toward w_1, the ratio $\frac{z_1}{z_2}$ decreases gradually from 0.6928 to 0.3728, while $\frac{z_3}{z_4}$ increases consistently from 0.1511 to 0.4711, indicating controlled sensitivity to weight adjustments. In contrast, the weighted sum method exhibits abrupt changes, with $\frac{z_1}{z_2}$ rising sharply and $\frac{z_3}{z_4}$ declining drastically, leading to dominance of one objective over the other. Under equal-preference conditions, the proposed model produces feasible and interpretable values, whereas the weighted sum approach fails to generate a solution. Overall, the findings confirm that the proposed framework offers smoother trade-off management and better objective preservation, making it more reliable for multi-objective fractional transportation problems.

Example 2:

A multi-objective fractional solid transportation problem involving three distinct objective functions is considered.

Table 1: Numerator of the first objective function.

Routing	R_1			R_1			R_1			8
		R_2			R_2			R_2		16
			R_3			R_3			R_3	18
	DR_1			DR_2			DR_3			Source capacity
SC_1	4	9	2	1	14	6	10	4	15	12
SC_2	15	7	12	11	5	13	8	3	12	15
SC_3	10	3	8	9	7	2	6	14	11	15
Destination requirement	10			14			18			42

Table 2: Denominator of the first objective function.

Routing	R_1			R_1			R_1			8
		R_2			R_2			R_2		16
			R_3			R_3			R_3	18
	DR_1			DR_2			DR_3			Source capacity
SC_1	7	19	4	10	3	16	17	1	13	12
SC_2	12	23	2	6	14	21	18	22	24	15
SC_3	15	8	25	5	11	20	9	8	7	15
Destination requirement	10			14			18			42

Table 3: Numerator of the second objective function.

Routing	R_1			R_1			R_1			8
		R_2			R_2			R_2		16
			R_3			R_3			R_3	18
	DR_1			DR_2			DR_3			Source capacity
SC_1	11	4	16	14	10	3	11	9	7	12
SC_2	7	13	2	12	8	5	13	4	16	15
SC_3	9	18	6	1	17	15	2	10	18	15
Destination requirement	10			14			18			42

Table 4: Denominator of the second objective function.

Routing	R_1			R_1			R_1			8
		R_2			R_2			R_2		16
			R_3			R_3			R_3	18
	DR_1			DR_2			DR_3			Source capacity
SC_1	6	13	2	5	10	14	13	2	6	12
SC_2	9	15	4	3	12	8	15	4	10	15
SC_3	11	7	17	1	16	9	7	12	17	15
Destination requirement	10			14			18			42

Table 5: Numerator of the third objective function.

Routing	R_1			R_1			R_1			8
		R_2			R_2			R_2		16
			R_3			R_3			R_3	18
	DR_1			DR_2			DR_3			Source capacity
SC_1	7	2	10	9	5	3	8	6	9	12
SC_2	4	13	6	12	7	2	1	11	5	15
SC_3	8	1	11	10	4	13	3	12	7	15
Destination requirement	10			14			18			42

Table 6: Denominator of the third objective function.

Routing	R_1			R_1			R_1			8
		R_2			R_2			R_2		16
			R_3			R_3			R_3	18
		DR_1			DR_2			DR_3		Source capacity
SC_1	14	3	11	6	12	15	20	16	8	12
SC_2	8	17	5	9	1	18	14	5	10	15
SC_3	19	2	10	4	7	13	3	19	12	15
Destination requirement		10		14			18			42

Solution:

For each objective $f_k(x)$, formulate the corresponding single-objective optimization problem, apply the SDR-based algorithm separately to each single-objective model and compute the optimal solution $x^{(k)}$.

The optimum solution for the first objective function is $\frac{285}{276}$.

The optimum solution for the second objective function is $\frac{303}{304}$.

The optimum solution for the third objective function is $\frac{110}{140}$.

Define normalized deviations for each objective: $D_k(x) = \frac{f_k(x)-f_k^{min}}{f_k^{max}-f_k^{min}}$, k=1,…,m.

$Z_1 = 285; Z_2 = 276; Z_3 = 303; Z_4 = 304; Z_5 = 110; Z_6 = 140$

${Z_1}^{min} = 100$ and ${Z_2}^{max} = 315$.

$$Z_1 = 285 \quad \frac{285-100}{315-100} = 0.8604 \qquad Z_4 = 304 \quad \frac{304-100}{315-100} = 0.9488$$

$$Z_2 = 276 \quad \frac{276-100}{315-100} = 0.8186 \qquad Z_5 = 110 \quad \frac{110-100}{315-100} = 0.0465$$

$$Z_3 = 303 \quad \frac{303-100}{315-100} = 0.9530 \qquad Z_6 = 140 \quad \frac{140-100}{315-100} = 0.1860$$

$\frac{Z_1}{Z_2} = \frac{0.8604}{0.8186} = 1.0510; \frac{Z_3}{Z_4} = \frac{0.9530}{0.9488} = 0.0044; \frac{Z_5}{Z_6} = \frac{0.0465}{0.1860} = 0.2500$

The MOFSTP is reformulated as a unified single-objective model by optimizing a collective deviation measure, in which a common deviation factor is modulated based on the priority weights assigned to the individual objectives.

Table 7: Comparative Performance of Proposed SDR Method and Weighted Sum Method under Different Weight Assignments

Weights Assigned **(w_1, w_2, w_3)**	**Proposed method**	**Weighted sum method**				
	$\frac{Z_1}{Z_2}$	$\frac{Z_3}{Z_4}$	$\frac{Z_5}{Z_6}$	$\frac{Z_1}{Z_2}$	$\frac{Z_3}{Z_4}$	$\frac{Z_5}{Z_6}$
$w_1 = 0.1; w_2 = 0.9; w_3 = 0$	1.2580	1.0274	0.4800	0.1051	0.9039	0
$w_1 = 0.2; w_2 = 0.8; w_3 = 0$	1.2350	1.0504	0.4800	0.2102	0.8035	0
$w_1 = 0.3; w_2 = 0.7; w_3 = 0$	1.2120	1.034	0.4800	0.3153	0.7030	0
$w_1 = 0.4; w_2 = 0.; w_3 = 0.6$	1.1890	1.2344	0.5720	0.4204	0	0.1500
$w_1 = 0.5; w_2 = 0; w_3 = 05$	1.1660	1.2344	0.5950	0.5255	0	0.1250
$w_1 = 0.6; w_2 = 0; w_3 = 0.4$	1.1430	1.2344	0.6180	0.6306	0	0.1000
$w_1 = 0; w_2 = 0.3; w_3 = 0.7$	1.1430	1.1654	0.5490	0	0.3013	0.1750
$w_1 = 0; w_2 = 0.2; w_3 = 0.8$	1.1430	1.1884	0.5260	0	0.2008	0.2000
$w_1 = 0; w_2 = 0.1; w_3 = 0.9$	1.1430	1.2114	0.5720	0	0.1004	0.2250
$w_1 = 0.3; w_2 = 0.3; w_3 = 0.4$	1.2120	1.1654	0.6180	0.3153	0.3013	0.1000
$w_1 = 0.3; w_2 = 0.4; w_3 = 0.3$	1.2120	1.1424	0.6410	0.3150	0.4017	0.0750
$w_1 = 0.4; w_2 = 0.3; w_3 = 0.3$	1.1890	1.1654	0.6410	0.4204	0.3013	0.0750
Without Preference	1.0510	1.0044	0.2500	-	-	-

The tabulated results show that the proposed method preserves meaningful fractional values under all weight configurations. For weight settings such as (0.1,0.9,0), (0.2,0.8,0), and (0.3,0.7,0), the model maintains stable contributions—particularly keeping $\frac{Z_5}{Z_6}$ consistent—whereas the weighted sum approach significantly distorts or suppresses this objective. When emphasis shifts to combinations like (0.4,0,0.6) and (0.3,0.3,0.4), the proposed framework adjusts smoothly, producing balanced objective values, while the weighted sum method assigns negligible or zero impact to certain objectives. Overall, the findings indicate that the proposed technique consistently retains the influence of all fractional objectives across varying preferences, demonstrating greater robustness than the conventional weighted sum method.

CONCLUSION

This study presents a structured two-phase normalized deviation algorithm for solving multi-objective fractional solid transportation problems. By integrating SDR-based decomposition with a unified deviation framework, the proposed method avoids the limitations of traditional aggregation techniques and provides a computationally efficient solution mechanism. The approach ensures balanced trade-offs among conflicting objectives while preserving Pareto efficiency. Numerical results confirm that the method yields stable and interpretable solutions under varying preference structures. The proposed framework can be extended to fuzzy, stochastic, and large-scale transportation systems, offering promising directions for future research.

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